Stock Market Liberalization and the Cost of Capital in Emerging Markets *

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November 2007

*I would like to thank Viral Acharya, Suleyman Basak, Bernard Dumas, Francisco Gomes, Denis Gromb, Kenn Judd, Anna Pavlova, Raman Uppal, Paolo Volpin, Joshua White, and seminar participants at the University of British Columbia, Kellogg, London Business School, London School of Economics, University of Oxford (Said School), University of Colorado (Boulder), Tilburg, HEC Paris, Bocconi, Imperial College, Padova, Ente Luigi Einaudi, USS Lugano, and participants at various conferences. Bhamra: 2053 Main Mall, Vancouver BC, Canada V6T 1Z2; Email: harjoat.bhamra@sauder.ubc.ca.

Abstract

I study how stock market liberalization changes an emerging market's cost of capital. I do so in a Lucas economy with two dividend trees. One dividend tree represents the emerging market's dividends while the other tree represents the dividends paid by all other countries. I solve for equilibrium asset prices in two versions of the economy. In the first version, stock markets are partially liberalized, because the emerging market's residents cannot invest in foreign stock markets. All other agents are unconstrained. In the second version, stock markets are fully liberalized, because there are no investment constraints. I show that moving from partial to full liberalization causes an increase in the emerging market's cost of capital and risk premium, despite better international risk sharing.

JEL Classification Numbers: D52, F36, G11, G12, G15
Keywords: international finance, stock market integration, continuous time, general equilibrium, incomplete markets

Financial liberalization was once recommended very strongly by the IMF, World Bank and US Treasury, amongst other institutions.[1] By the 1980's and early 1990's, many emerging market countries had followed this recommendation. That is, they had opened their stock markets to foreign investors and allowed their own residents to invest abroad. By end of the 1990's severe financial crises had ravaged many of these countries.[2] But two countries in particular, China and India, avoided this upheaval. China and India had only *partially* liberalized their stock markets. They allowed some foreign investment, but placed severe barriers on their own residents investing abroad.[3] As the 2006 Nobel-Prize winner Stiglitz writes [see p.34, Stiglitz (2006)], "Even as they opened up their markets for long-term investment, the two Asian giants—India and China—have restricted short-term capital flows." In contrast, other countries allowed both foreign investment and outward investment by domestic residents, and subsequently faced financial turmoil. Is it possible that partial liberalization, as pursued by China and India, has different effects on the cost of capital and stock-market risk premia, compared with the more full-blooded liberalization pursued by other countries? That is the question studied in this paper.

The argument for the removal of barriers to cross-border portfolio flows relies on the following simple intuition. Removing barriers to international portfolio flows improves risk sharing, which reduces risk premia and hence, the cost of capital.[4] And, a lower cost of capital leads to increased investment, which is beneficial. This line of reasoning suggests that the policies of China and India are suboptimal in the

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[1]I use the term financial liberalization to refer to capital account liberalization. In 1997, the IMF went so far as to formally propose amending its articles to incorporate financial liberalization, not merely as a recommendation, but as an explicit raison d'etre. The following quote is taken from the Statement of the Interim Committee on the Liberalization of Capital Movements under an Amendment of the Articles, September 21, 1997: '...the Committee invites the Executive Board to complete its work on a proposed amendment of the Fund's Articles that would make the liberalization of capital movements one of the *purposes* of the Fund [emphasis added] '.

[2]For instance, the Asian crisis of 1997 was followed by the Russian crisis in 1998 and the Brazilian crisis in 1999.

[3]Until very recently, Chinese residents could not invest in shares listed outside of China. Even with the gradual introduction of the Qualified Domestic Institutional Investor's (QDII) program in 2006, Chinese residents still face restrictions on the amounts they can invest in foreign stocks. Foreigners investing in Chinese shares have historically faced less severe restrictions. Since 1992, Chinese firms have been able to issue foreign currency-denominated shares for purchase solely by non-residents. In 1993, Qingdao Beer was the first mainland Chinese firm to list its shares on the Hong Kong Stock Exchange (see Prasad and Wei (2005) and Cheung, Chow, Chang, and Li (2006)). Indian residents cannot invest in foreign equities—this was the case historically and continues to be the case. Foreign institutional investors (FII's) can buy limited amounts of shares in Indian companies. Over time, the restrictions faced by FII's have been reduced (see Shah and Patnaik (2007)).

[4]Henry (2003) gives a partial equilibrium argument based on Stulz (1999).

sense that full liberalization (no constraints on any investor) instead of just partial liberalization (residents of China/India cannot invest in foreign stock markets, but foreigners are unconstrained) would lead to a greater reduction in the cost of capital. But Stiglitz (2002) thinks differently: 'Capital-market liberalization is inevitably accompanied by huge volatility .... It increases the risks of investing in the country, and thus, investors demand a risk-premium.' This view challenges the standard intuition.

To get to the heart of the economics underlying this debate, I study a general equilibrium exchange economy consisting of an 'emerging market country' and the 'rest of the world'. Agents are fully rational and there are no information asymmetries. To compare the cost of capital in the emerging market as one goes from partial to full liberalization, I consider two versions of the economy. In both versions, all agents have identical preferences and face no constraints on risk-free borrowing and lending inside their own countries and across borders.[5] But the extent to which stock markets are integrated internationally differs as follows:

1. Partial liberalization—agents from the emerging market can invest in the emerging market country's stock market, but they *cannot* invest in stock markets in the rest of the world. Agents from the rest of the world can invest in *all* stock markets, including the emerging market country's stock market.[6]

2. Full liberalization—agents face no constraints on stock market investment.[7]

Observe that under partial liberalization markets are incomplete and risk sharing is imperfect. In contrast, under full liberalization markets are complete and risk sharing is perfect.

My main result shows that moving from partial to full liberalization *increases* the cost of capital in the emerging market country. At first blush, this result may seem counterintuitive, because it says that under partial liberalization, when risk sharing is imperfect, the cost of capital is lower than under full liberalization, where

[5]While Chinese individuals cannot invest in foreign risk-free securities, the Chinese government can and does. To ensure my analysis does not ignore this, I assume there are no constraints on risk-free borrowing and lending across borders.

[6]Errunza and Losq (1985) refer to this form of partial liberalization as mild segmentation.

[7]Errunza and Losq (1985) refer to full liberalization as complete integration.

risk sharing is perfect. I now explain why removing barriers to investment in foreign markets can lead to a higher cost of capital in the emerging market.

The cost of capital for investing in the emerging market, $\mu_{e,t}$, is the risk-free rate, r_t, plus the risk premium, i.e.

$$\mu_{e,t} = r_t + \frac{1}{dt} Cov_t(dR_{e,t}, dR_{W,t}), \tag{1}$$

where the standard expression for the risk premium is $\frac{1}{dt} Cov_t(dR_{e,t}, dR_{W,t})$, the covariance (per unit time) between stock returns in the emerging market, $dR_{e,t}$, and the world market, $dR_{W,t}$.[8] Both the risk-free rate and the covariance term are higher under full liberalization than partial liberalization. Since consumption risk sharing is better under full liberalization than partial liberalization, demand for precautionary savings is lower under full liberalization. Therefore moving from partial to full liberalization increases the risk-free rate. Also, under full liberalization, stock markets are more integrated internationally, which increases the covariance of the emerging market's stock returns with the world market return.

To understand why the covariance increases, I note that the covariance term is the variance of world market returns multiplied by the sum of the emerging market's discount-rate and cash-flow betas. World market returns will not be significantly affected by what happens when an emerging market allows its residents to invest abroad in risky securities. Also, dividends are exogenous. Therefore, moving from partial to full liberalization can only change the emerging market's discount-rate beta. The emerging market's discount-rate beta increases if the emerging market's price-dividend ratio becomes more procyclical, which occurs if the emerging market's discount rate becomes more countercyclical.

The discount rate is the risk-free rate adjusted for risk. Under partial liberalization, the risk-free rate contains a procyclical term. A positive shock to world output growth increases the consumption level of the emerging market's residents. But they consume a smaller *share* of world output, because their investment opportunity set is worse than residents in other countries. Markets are incomplete only because the emerging market's residents are constrained. As their consumption share falls, the

[8] The $\frac{1}{dt}$ term appears because the expression for $\mu_{e,t}$ is in continuous time. The general form of the risk-return relation for the emerging market is identical under both partial and full liberalization, a result first derived in Errunza and Losq (1985).

impact of market incompleteness on prices falls. Reducing the impact of market incompleteness means risk sharing improves. A positive shock to world output growth thus improves risk sharing, which reduces demand for precautionary savings. Therefore the risk-free rate and discount rate are higher when there is a positive shock to world output growth. In short, the discount rate contains a procyclical component, because of the way changes in the cross-sectional consumption distribution impact demand for precautionary savings.

Under full liberalization markets are complete and agents are homogeneous, and so the cross-sectional consumption distribution is irrelevant for asset prices. The procyclical component in the risk-free rate and hence the discount rate vanishes. Therefore, moving from partial to full liberalization makes the emerging market's discount rate more countercyclical. That increases the emerging market's discount-rate beta. The emerging market's risk premium is thus higher under full liberalization.

This paper also makes a methodological contribution by providing a general method of finding closed-form solutions for asset prices in an economy with more than one dividend tree, incomplete markets, and heterogeneous agents.[9] To be successful, the methodology I use must deal with the following three complications. First, introducing a second dividend tree gives rise to a new state variable for stock prices: the share of world dividends produced by one of the dividend trees. Second, agent heterogeneity and market incompleteness add *another* state variable: the cross-sectional wealth distribution across agents. Third, the cross-sectional wealth distribution depends on stock prices, because the wealth of all agents is affected by risky stock prices.[10] Thus, there is no clean separation between stock prices and one of their state variables (the cross-sectional wealth distribution), and so, prices and the wealth distribution must be solved for *jointly*.[11] The above trio of complications

[9]The version of the economy in which I exploit this methodology is, of the course, the version where stock markets are partially liberalized.

[10]When stock markets are partially liberalized, residents of the emerging market are the only agents who are constrained and even they can invest in a risky asset: their own stock market.

[11]Formally, the price of a single stock satisfies a forward-backward stochastic differential equation (FBSDE) system. The backward stochastic differential equation for the stock price depends on one agent's share of aggregate wealth (the cross-sectional wealth distribution), which satisfies a forward stochastic differential equation. This forward stochastic differential equation depends on the stock price. Thus the forward and backward stochastic differential equations are coupled into a FBSDE system. Ma and Yong (1999) summarize applications of FBSDE systems in finance. See Schroder and Skiadas (1999), Schroder and Skiadas (2003), Schroder and Skiadas (2005) and

does not arise in previous general-equilibrium exchange economy models with closed-form solutions. For example, in Cochrane, Longstaff, and Santa-Clara (2007) there is more than one dividend tree, but markets are complete.[12] And in Basak and Cuoco (1998), while markets are incomplete and agents are heterogeneous, there is only one dividend tree and the cross-sectional wealth distribution does not depend on prices. Nevertheless, in a model with two dividend trees *and* incomplete markets with heterogeneous agents, I can still characterize security prices in closed form, albeit approximately.[13] I do this using the method of matched asymptotic expansions.[14] To the best of my knowledge, the application of this solution method is novel in the finance literature.

There is a large body of work that examines the effects of international risk sharing on prices within a static framework. See, for instance, Black (1974), Stulz (1981), Errunza and Losq (1985), Eun and Janakiramanan (1986), Errunza and Losq (1989) and Errunza, Losq, and Padmanabhan (1992). Of these papers, the only ones which derive the risk-return relation for the emerging market country's stock market under partial liberalization are Errunza and Losq (1985) and Errunza and Losq (1989). These two papers consider a static framework and do not derive asset prices in terms of the economy's underlying state variables. The risk-return relation for the emerging market country's stock market does not change as one moves from partial to full liberalization. Without expressions for asset prices in

Schroder and Skiadas (2007) for applications to portfolio choice and Basak and Gallmeyer (2003) for an equilibrium asset pricing application involving taxation.

[12]Menzly, Santos, and Veronesi (2004) also derive asset prices in an economy with more one dividend tree and complete markets. But Menzly, Santos, and Veronesi assume dividend growth for an individual stock follows a particular non i.i.d. process, chosen specially for tractability, whereas both this paper and Cochrane, Longstaff, and Santa-Clara (2007) assume dividend growth is normal i.i.d.

[13]The closed-form solutions obtained in Menzly, Santos, and Veronesi (2004) with many dividend trees and complete markets are also approximate.

[14]I obtain a 2-dimensional elliptic partial differential equation (pde) for the emerging market stock-price/world-output ratio by using the Feynman-Kac Theorem. When the correlation in output growth between the emerging market and the rest of the world is unity, markets are complete despite constraints, and prices can be obtained exactly in closed form. I expand around this base case solution in a Taylor series. But this naive approach provides an expansion, which satisfies only 3 of the 4 boundary conditions for the pde. To modify this expansion so it satisfies all the boundary conditions, I employ the 'method of matched asymptotic expansions'—see Hinch (1991) and Kevorkian and Cole (1996) for textbook treatments. Essentially this involves solving the partial differential equation in the region adjacent to the boundary, where the existing expansion is invalid as a solution, and obtaining a new expansion. The new expansion contains arbitrary functions of one of the state variables, which are chosen so that away from the boundary the new expansion matches up with the old expansion. Thus, one obtains a solution which satisfies all 4 boundary conditions.

terms of the state variables, Errunza and Losq cannot study how moving from partial to full liberalization changes the risk premium and cost of capital in the emerging market. Dynamic models studying the impact of international risk sharing on asset pricing include Sellin and Werner (1993) and Pavlova and Rigobon (2007).[15] Sellin and Werner (1993) study constraints on cross-border investment, but work in a production economy with constant returns to scale, where the only endogenous price is the risk-free rate. Pavlova and Rigobon study the impact of portfolio constraints on agents in a large developed economy, such as the US or the EU on the comovement of stock prices and the terms of trade, whereas this paper studies the impact on the cost of capital.[16]

There is an empirical literature, which focuses on the effect of liberalizing markets on asset prices and returns (see for example, Bekaert and Harvey (1997), Bekaert and Harvey (2000), Kim and Singal (2000), Henry (2000) and Chari and Henry (2004)). But this empirical literature is limited by previous theoretical work, which focuses solely on the effects of full liberalization versus autarky. In this paper, I seek to broaden the results of asset pricing theory available to empiricists by including the intermediate case of partial liberalization.

In a domestic framework, where there is only one stock, many papers have studied the impact of limited risk sharing on various aspects of aggregate market prices.[17] By using a framework with two dividend/output processes and thus two stock markets, I can study the impact of limited risk sharing in a smaller emerging market country, which forms part of the world economy. The analysis is undertaken in a general equilibrium setting, where to ensure that the emerging market country is not unreasonably large, I only assume that less than half of world output is produced in the emerging market country.

[15] Chaieb and Errunza (2007) extend Errunza and Losq (1985) to a dynamic framework with deviations from PPP, but do not solve for asset prices in terms of the economy's state variables.

[16] I do not focus on the impact of international risk sharing on welfare. See, for example, Subrahmanyam (1975) and Errunza and Losq (1989) who employ a static framework. Obstfeld (1994), Tesar (1995), Lewis (1996), Basak (1996), Devereux and Saito (1997), Dumas and Uppal (2001) and Basak and Croitoru (2007) conduct dynamic analyses.

[17] See Telmer (1993), Lucas (1994), Heaton and Lucas (1996), Zapatero (1998), Basak and Cuoco (1998), Citanna and Schmedders (2005), Gallmeyer and Hollifield (2007) and Bhamra and Uppal (2007).

I The Model

In this section, I describe how I model the world's economy. I state my assumptions about preferences, dividends, financial assets and portfolio constraints. Then, I describe the optimization problem agents face and the equilibrium concept I use.

I work in a continuous-time, infinite-horizon, pure-exchange economy with perfect information. The world consists of two regions, an emerging market country (Region e) and all other countries of the world (Region o). There is one perishable consumption good, and in each region there is an exogenous supply of this good, i.e. a dividend tree. Agents have identical preferences within and across regions.

Agents can borrow and lend freely to each other at the risk-free rate. I consider two separate stock-market structures:

1. Partial liberalization—agents from the emerging market (Region e) can invest in the emerging market country's stock market, but cannot invest in foreign (Region o) stock markets. Agents from the rest of the world (Region o) can invest in all stock markets (Regions e and o).

2. Full liberalization—agents face no constraints on investing in domestic and foreign markets.

In order to focus solely on the impact of stock market liberalization, my model abstracts away the effects of foreign-exchange rates and heterogeneity in preferences. Observe that the model differs from Lucas (1978) in two respects. First, there are two dividend trees and therefore two risky assets. Second, in the version of the economy with partially liberalized stock markets, financial markets are incomplete and agents are heterogeneous with respect to their investment opportunities.

I.A Preferences, Dividends and Financial Assets

All agents in the world have identical preferences. The Region i representative agent, hereafter referred to as Agent i, has utility

$$U_{i,t} = E_t \left[\int_t^{\infty} e^{-\beta(s-t)} \ln C_{i,s} ds \right], \; i \in \{e, o\}, \tag{2}$$

where β is the rate of time preference and C_i is her consumption.

In each region, the consumption good is produced by a dividend tree.[18] In Region i, the dividend tree produces output Y_i, given by

$$\frac{dY_{i,t}}{Y_{i,t}} = \mu_Y dt + \sigma_Y dZ_{Y,i,t},\ i \in \{e, o\}, \tag{3}$$

where $Z_{Y,i,t}$, $\{e, o\}$ are Brownian motions, such that $dZ_{Y,o,t} dZ_{Y,r,t} = \rho dt$, where the constant ρ denotes the cross-region correlation in dividend growth rates. The conditional expected growth rate of output and the conditional volatility of output growth, μ_Y and σ_Y, respectively, are constant and the same in each region.[19] It will be useful to define $s_{i,t} = \frac{Y_{i,t}}{Y_t}$ as the share of world output produced in Region i.

There is a locally risk-free bond in zero-net supply, with the risk-free interest rate r_t. The stock market in Region i is the claim to Region i's dividend tree, and has price S_i. The total instantaneous return on Region i's stock market is $R_{i,t}$, where

$$dR_{i,t} = \frac{dS_{i,t} + Y_{i,t} dt}{S_{i,t}} = \mu_{i,t} dt + \sigma_{i,t} dZ_{R,i,t},\ i \in \{e, o\}, \tag{4}$$

and $Z_{R,i,t}$, $i \in \{e, o\}$ are Brownian motions.[20] The cross-region correlation in stock market returns is $\rho_{eo,t}$, where $dZ_{R,e,t} dZ_{R,o,t} = \rho_{eo,t} dt$.[21] Since the locally risk-free bond is in zero-net supply, the value of the world market portfolio, $S_{W,t}$, is the combined value of regional stock markets, i.e. $S_{W,t} = S_{e,t} + S_{o,t}$. The total instantaneous return on the world market portfolio is $R_{W,t}$, where

$$dR_{W,t} = \frac{dS_{W,t} + Y_t dt}{S_{W,t}} = \mu_{W,t} dt + \sigma_{W,t} dZ_{W,t}, \tag{5}$$

where $Z_{W,t}$ is a Brownian motion.[22]

[18] In this framework, dividends and output are of course synonyms for each other

[19] This assumption is made for algebraic simplicity. It can be relaxed without changing the main results.

[20] The Brownian motions driving return shocks, $Z_{R,i,t}$, $i \in \{e, o\}$ are of course related to the Brownian motions driving output growth shocks, $Z_{Y,i,t}$, $i \in \{e, o\}$. In general, $Z_{R,i,t}$ will not equal $Z_{Y,i,t}$, because price-dividend ratios can be stochastic.

[21] Note that $\rho_{eo,t}$ differs from ρ. Whereas, ρ is the exogenously given cross-region correlation in output growth, $\rho_{eo,t}$ is the cross-region correlation in stock returns, which is determined endogenously in equilibrium.

[22] Note that because $dR_{W,t} = \frac{S_{e,t}}{S_{W,t}} dR_{e,t} + \frac{S_{o,t}}{S_{W,t}} dR_{o,t}$, $Z_{W,t}$ is related to $Z_{R,i,t}$, $i \in \{e, o\}$, by $Z_{W,t} = \frac{S_{e,t}\sigma_{e,t}}{S_{W,t}\sigma_{W,t}} Z_{R,e,t} + \frac{S_{o,t}\sigma_{o,t}}{S_{W,t}\sigma_{W,t}} Z_{R,o,t}$.

The quantities r_t, $S_{e,t}$, $S_{o,t}$, $\mu_{e,t}$, $\mu_{o,t}$, $\sigma_{e,t}$, $\sigma_{o,t}$ and $\rho_{eo,t}$ are endogenous and will be determined in equilibrium. Their values will depend on whether there is partial or full liberalization.

I.B Agents' Optimization Problems

Agent i is assumed to have initial wealth $\overline{X}_i$ and initial portfolio proportions $\overline{\pi}_{ij}$, $j \in \{e, o\}$, where $\overline{\pi}_{ij}$ is the proportion of Agent i's wealth initially invested in Region j's stock market. The problem of each agent is to maximize lifetime utility $U_{i,0}$ in (2) subject to a static budget constraint. The budget constraint requires that the present value of all future consumption is no more than the initial wealth with which each agent is endowed:

$$E_0 \left[\int_0^\infty \frac{\xi_{i,t}}{\xi_{i,0}} C_{i,t} dt \right] \leq \overline{X}_i, \tag{6}$$

in which $\xi_{i,t}$ is the marginal utility of Agent i at date t (also known as the state-price density or stochastic discount factor). The first-order condition is

$$\kappa_i \frac{\xi_{i,t}}{\xi_{i,0}} = \frac{\partial \ln C_{i,t}}{\partial C_{i,t}} = e^{-\beta t} C_{i,t}^{-1}, \tag{7}$$

in which κ_i is the Lagrange multiplier on the static budget constraint in (6). The process for $\xi_{i,t}$ is given by (see Duffie (2001, Section 6.D, p. 106)):

$$\frac{d\xi_{i,t}}{\xi_{i,t}} = -r_{i,t} dt - \theta_{i,t} dZ_{\xi_i,t}, \tag{8}$$

in which $r_{i,t}$ is the risk-free interest rate, $\theta_{i,t}$ is the market price of risk and $Z_{\xi_i,t}$ is a standard Brownian motion defined in (A44).

In the partial liberalization version of the economy, markets are incomplete because Agent e cannot invest in Region o's stock market. But there are no constraints on risk-free borrowing and lending. Therefore, both agents face the same risk-free rate, i.e. $r_{e,t} = r_{o,t} = r_t$.

However, the market prices of risk are different for Agents e and o, because they face different investment opportunity sets. The only risky asset Agent e can invest in is the stock market for Region e, so her market price of risk equals the Sharpe ratio for Region e's stock market, i.e.

$$\theta_{e,t} = \phi_{e,t}, \tag{9}$$

where

$$\phi_{i,t} = \frac{\mu_{i,t} - r_t}{\sigma_{i,t}},\ i \in \{e, o\}, \tag{10}$$

is the Sharpe ratio for Region i's stock market. Agent o is unconstrained, so her market price of risk depends on the Sharpe ratios of both regional stock markets, i.e.

$$\theta_{o,t} = \frac{1}{1 - \rho_{eo,t}^2} \left(\phi_{e,t}^2 + 2\rho_{eo,t}\phi_{e,t}\phi_{o,t} + \phi_{o,t}^2 \right)^{1/2}. \tag{11}$$

Under full liberalization, markets are complete, so all agents face the same the risk-free rate and market price of risk. All agents can invest in both stock markets. The market price of risk is then given by the right-hand side of (11).[23]

II The Cost of Capital in the Emerging Market

In this section, I derive a general expression for the cost of capital in the emerging market (Region e), which is valid under both partial and full liberalization. The cost of capital is the sum of the risk-free rate and a risk premium. It is well known (see Cochrane (2005, Section 1.5, p. 29)) that under both partial and full liberalization the risk-free rate, r_t, is given in terms of the equilibrium state-price density, ξ_t, by

$$r_t = -\frac{1}{dt} E_t \left[\frac{d\xi_t}{\xi_t} \right]. \tag{12}$$

It remains to obtain a corresponding expression for the emerging market's risk premium.

I start by deriving the optimal portfolio demands of both agents. I then substitute optimal portfolio demands into two market clearing equations, one for each stock market, which I solve simultaneously to obtain the regional stock market risk premia, $\mu_{e,t} - r_t$ and $\mu_{o,t} - r_t$.[24] The resulting emerging market (Region e) risk premium is given in the lemma below.

[23]Starting on p.24 of the Appendix I give a rigorous derivation of (9) and (11), by using the duality techniques of Cvitanic and Karatzas (1992).

[24]The expressions for the risk premium in Region o under partial and full liberalization are given in (A37) and (A39), respectively.

Lemma 1 *Under both partial and full liberalization the emerging market's risk premium is given by*

$$\mu_{e,t} - r_t = \frac{1}{dt} Cov_t(dR_{e,t}, dR_{W,t}), \tag{13}$$

where $dR_{W,t}$ is the return on the world market portfolio.

The above lemma tells us that moving from partial to full liberalization does not change the form of the risk-return relationship in the emerging market.[25] Coupled with the fact that the right-hand side contains only endogenous quantities, the invariance of (13) makes it difficult to use in gaining a deeper understanding of the economic forces at play when the emerging market's risk premium changes. To make any progress, one must rewrite (13), as far as possible, in terms of exogenous quantities. I do this in two steps.

First, I rewrite (13) as

$$\mu_{e,t} - r_t = \beta_{e,t} \sigma^2_{W,t}, \tag{14}$$

where $\beta_{e,t}$ is the emerging market's beta,[26] defined by

$$\beta_{e,t} = \frac{Cov_t(dR_{e,t}, dR_{W,t})}{Var_t(dR_{W,t})} \tag{15}$$

and $\sigma_{W,t}$ is the volatility of the world-market portfolio return in (5). The value of the world market portfolio is the combined value of world stock markets, $S_{W,t} = S_{e,t} + S_{o,t}$, which is given by (see p.26 of the Appendix for a proof)

$$S_{W,t} = \frac{Y_t}{\beta}. \tag{16}$$

Therefore, the return on the word market portfolio is

$$\frac{dR_{W,t}}{R_{W,t}} = \frac{dY_t}{Y_t} + \beta dt, \tag{17}$$

which implies that the volatility of the world-market portfolio return equals world-output growth volatility, i.e. $\sigma_{W,t} = \sigma_Y$. The sole endogenous quantity in the right-hand side of (14) is thus the emerging market's beta.

[25]Errunza and Losq (1985) also find that the risk-return relationship in the emerging market is the same under partial and full liberalization—see their Equation (2).

[26]The emerging market beta, β_e, is not to be confused with the rate of time preference, β.

Second, I break the emerging market's beta into two components: a cash-flow beta, reflecting news about the emerging market's fundamentals and a discount-rate beta, reflecting news about the emerging market's discount rate. I define the cash-flow beta as

$$\beta_{e,t}^{CF} = \frac{Cov_t\left(\frac{dY_{e,t}}{Y_{e,t}}, dR_{W,t}\right)}{Var_t(dR_{W,t})} \tag{18}$$

and the discount-rate beta as

$$\beta_{e,t}^{DR} = \frac{Cov_t\left(\frac{dp_{e,t}}{p_{e,t}}, dR_{W,t}\right)}{Var_t(dR_{W,t})}, \tag{19}$$

where $p_{e,t} = \frac{S_{e,t}}{Y_{e,t}}$ is the emerging market's price-dividend ratio.[27]

To understand why the price-dividend ratio appears in (19), note that intuitively, one would expect the price-dividend ratio to increase when there is a negative shock to the emerging market's discount rate.

In the Technical Appendix to this paper I verify the above intuition is correct by extending the Gordon growth model to the case of a time-varying discount rate, defined by the stochastic process

$$k_{e,t} = r_t + \rho_{\xi,e,t}\theta_t\sigma_Y, \tag{20}$$

which is the sum of the risk-free rate and a risk adjustment. The risk adjustment is the product of the size of the correlation between shocks to the equilibrium state-price density and the emerging market's output growth, $\rho_{\xi,e,t}$, the equilibrium market price of risk, θ_t, and the emerging market's output growth volatility, σ_Y.[28]

[27] Campbell and Vuolteenaho (2004) use a two-beta CAPM to explain the size and value anomalies. Their decomposition of a stock's beta into cash flow and discount-rate beta differs from this paper. The difference is that Campbell and Vuolteenaho (2004) decompose the emerging market's beta by splitting the return on the world market portfolio into cash flow and discount rate components, whereas I split the emerging market's return into cash flow and discount rate components.

[28] The risk adjustment in the emerging market's discount rate does not equal its risk premium. This is an important point, because it implies that it is not tautological to understand the risk premium in terms of the behavior of the discount rate. To see why the point is true, note that the emerging market's risk premium can be expressed as $-Cov_t\left(\frac{d\xi_t}{\xi_t}, dR_{e,t}\right)$, while the risk adjustment in the emerging market's discount rate is $\rho_{\xi,e,t}\theta_t\sigma_Y dt = -Cov_t\left(\frac{d\xi_t}{\xi_t}, \frac{dY_{e,t}}{Y_{e,t}}\right)$. Since $dR_{e,t} = \frac{dY_{e,t}}{Y_{e,t}} + \frac{dp_{e,t}}{p_{e,t}} + \frac{dY_{e,t}}{Y_{e,t}}\frac{dp_{e,t}}{p_{e,t}}$, the risk premium and the risk adjustment will be equal if and only if the price-dividend ratio does not covary with the state-price density.

It follows from Ito's Lemma that the cash-flow beta and the discount-rate beta add up to the total beta:

$$\beta_{e,t} = \beta_{e,t}^{CF} + \beta_{e,t}^{DR}. \tag{21}$$

The following proposition then follows immediately from (14).

Proposition 1 *Under both partial and full liberalization, the emerging market's risk premium is given by*

$$\mu_{e,t} - r_t = \beta_{e,t}^{CF} \sigma_{W,t}^2 + \beta_{e,t}^{DR} \sigma_{W,t}^2, \tag{22}$$

where $\sigma_{W,t}$ is the volatility of the return on the world market portfolio.

Equation (17) implies that the discount-rate beta is the only endogenous quantity in the right-hand side of (22). Therefore, the above proposition tells us that moving from partial to full liberalization increases the emerging market's risk premium if the emerging market's *discount-rate beta* increases.

The discount-rate beta increases when the covariance of shocks to world output growth with shocks to the price-dividend ratio is more positive. Because the price-dividend ratio falls when the discount rate rises, it follows that the discount-rate beta is higher when the covariance of shocks to the discount rate with shocks to world output growth is *more negative.* In other words, the discount-rate beta and hence the risk premium are higher when the discount rate is *more countercyclical* (with respect to world output growth).

In summary, to determine how moving from partial to full liberalization changes the emerging market's cost of capital one must understand the following:

- how the locally risk-free rate, r_t, changes, and
- how the cyclicality of the emerging market's discount rate, $k_{e,t}$, changes.

To determine how the above quantities change, I must solve for equilibrium asset prices under both partial and full liberalization. In the full liberalization version of the economy, there are no portfolio constraints, so agents are homogeneous and markets are complete. Thus, the full liberalization version of the economy is identical to Cochrane, Longstaff, and Santa-Clara (2007). Cochrane, Longstaff, and Santa-Clara (2007) derive all asset prices in closed form. In the next section I derive asset prices in the partial liberalization version of the economy and compare them with the full liberalization asset prices derived in Cochrane, Longstaff, and Santa-Clara (2007).

It is important to be aware that solving for asset prices is actually necessary. By considering the stock-market clearing condition for the emerging market

$$S_{e,t} = X_{e,t}\pi_{ee,t} + X_{o,t}\pi_{oe,t}, \tag{23}$$

it is not obvious what will happen to the emerging market's stock price upon moving from partial to full liberalization. While one knows that the proportion of their wealth that emerging residents invest in their own stock market, $\pi_{ee,t}$, will fall, it is not clear what will happen to their financial wealth, $X_{e,t}$. Neither is it clear what will happen to the financial wealth of residents in other countries, $X_{o,t}$, or the proportion of their wealth that they invest in the emerging market's stock market. Thus, one cannot use simple supply and demand arguments to find out how moving from partial to full liberalization affects the emerging market's stock price and cost of capital. One is left with no option but to solve the model fully.[29]

III Partial Liberalization and the Cost of Capital in the Emerging Market

I solve for equilibrium prices in the version of the economy, where the emerging-market country's stock market is partially liberalized. Thus, residents of the emerging market country cannot invest in foreign stock markets, but non-residents can invest in the emerging country's stock market, in addition to their own stock markets. Financial markets are incomplete and risk sharing is imperfect.

The equilibrium concept I use is a simple extension of Lucas (1978). Both agents maximize their expected lifetime utility and all markets must clear. Hence, in equilibrium, the two agents consume all of world output and their aggregate holding of the risk-free bond must be zero. Moreover, the two agents together must own all shares in both stock markets, subject to the constraint that Agent e cannot hold shares in foreign stock markets. The constraint renders markets incomplete. I can still derive equilibrium prices using a social planner, but she will have a stochastic

[29]Even if one assumes that moving from partial to full liberalization increases the financial wealth of emerging market residents at the expense of residents in other countries, the overall effect on the emerging market's stock price cannot be determined.

weight, λ_t (see Cuoco and He (1994)):

$$U_t = E_t \int_t^{\infty} e^{-\beta(s-t)} \left(\ln C_{o,s} + \lambda_s \ln C_{e,s}\right) ds. \tag{24}$$

Hence, the consumption sharing rule is given by

$$\frac{C_{o,t}^{-1}}{\lambda_t C_{e,t}^{-1}} = 1. \tag{25}$$

The above consumption sharing rule and market clearing for the consumption good, $C_{e,t} + C_{o,t} = Y_t$, imply that the stochastic weight has an intuitive interpretation in terms of agents' equilibrium shares of world consumption. Because Agent i's equilibrium share of world consumption, $\nu_{i,t}$, is defined by $\nu_{i,t} = \frac{C_{i,t}}{Y_t}$, it follows that $\nu_{o,t} = 1/(1+\lambda_t)$ and $\nu_{e,t} = 1 - \nu_{o,t}$. As in Basak and Cuoco (1998), the equilibrium state-price density, ξ_t, is given by the state-price density of the agent who faces complete markets, i.e. Agent o. Thus,

$$\xi_t = \xi_{o,t} = \kappa_o e^{-\beta t}(\nu_{o,t} Y_t)^{-1}. \tag{26}$$

Therefore, to determine equilibrium consumption in each region and hence the equilibrium state-price density, all one need do is find the stochastic weight, λ_t. From the equilibrium state-price density, one can use (12) to derive the equilibrium risk-free rate.

The following proposition gives the equilibrium risk-free rate under partial liberalization in terms of the equilibrium risk-free rate under full liberalization, together with exogenous parameters and the consumption sharing rule.[30]

Proposition 2 *The equilibrium risk-free rate under partial liberalization, r_t^P, is given in terms of the equilibrium risk-free rate under full liberalization, r_t^F, by*

$$r_t^P = r_t^F - \sqrt{1 - 2(1-\rho) s_{o,t} s_{e,t}}\, \rho_{\nu_o,Y,t} \sigma_{\nu_o,t} \sigma_Y, \tag{27}$$

where

$$r_t^F = \beta + \mu_Y - (1 - 2(1-\rho) s_{o,t} s_{e,t}) \sigma_Y^2, \tag{28}$$

[30]Under full liberalization, markets are complete and agents are homogeneous, so asset prices are determined by a single representative agent with logarithmic utility. Therefore, the equilibrium state-price density is $\xi_t = e^{-\beta t} Y_t^{-1}$. The risk-free rate under full liberalization then follows from (12). See also Cochrane, Longstaff, and Santa-Clara (2007) for a derivation.

$s_{e,t} = C_{e,t}/Y_t$ is the equilibrium share of world output consumed by Agent e, $s_{o,t} = 1 - s_{e,t}$, and $\rho_{\nu_o,Y,t}$ is the correlation between shocks to percentage changes in the consumption sharing rule $\nu_{o,t}$ and world output growth.

I use the superscripts P and F to distinguish between prices and returns under partial and full liberalization, respectively. The risk-free rate consists of three terms. The first is the rate of time preference, β, and the second, μ_Y, is the effect of intertemporal consumption smoothing. The third term is the precautionary savings term and is the only term which changes as one moves from partial to full liberalization. Because Agent o has better investment opportunities than Agent e, a shock to world output growth increases Agent o's optimal consumption more than Agent e's. Therefore, shocks to Agent o's optimal share of world output, the consumption sharing rule, $\nu_{o,t}$, are positively correlated with shocks to world output growth, i.e. $\rho_{\nu_o,Y,t} > 0$. Consequently, one can see from (27) that moving from partial to full liberalization increases the risk-free rate by decreasing the demand for precautionary savings. To see the intuition recall that under partial liberalization, risk sharing is imperfect, whereas under full liberalization, risk sharing is perfect. Moving from partial to full liberalization improves risk sharing in the economy, which reduces demand for precautionary savings and thereby raises the risk-free rate.

I now turn to the valuation of regional stock markets. Since the sum of stock market values across regions, $S_{W,t} = S_{o,t} + S_{e,t}$ is given in Equation (16), one need only determine the stock market value in one of the two regions. The emerging market's stock price is given by[31]

$$S_{e,t} = Y_{e,t} E_t \left[\int_t^{\infty} \frac{\xi_s Y_{e,s}}{\xi_t Y_{e,t}} ds \right]. \tag{29}$$

By substituting the equilibrium state-price density under partial liberalization given in (26) into (29) and rearranging, one obtains the following expression for the emerging market's stock price under partial liberalization:

$$S_{e,t}^P = Y_{e,t} \left(\nu_{o,t} p_{e,t}^F + \nu_{e,t} h_{e,t} \right), \tag{30}$$

[31]The technical condition for this to be true is that the discounted gains process, $G_{e,t} = \xi_t S_{e,t} + \int_0^t \xi_s Y_{e,s} ds$, must be a martingale under the natural measure $\mathbb{P}$. Using Ito's Lemma, it is straightforward to show that $G_{e,t}$ is a local martingale. But to prove the local martingale is a martingale, one would have to provide bounds for two endogenous quantities, stock-market return volatility in the emerging market and the equilibrium market price of risk.

where

$$p^F_{e,t} = E_t\left[\int_t^\infty e^{-\beta(s-t)}\frac{s_{e,s}}{s_{e,t}}ds\right], \tag{31}$$

and

$$h_{e,t} = E_t\left[\int_t^\infty e^{-\beta(s-t)}\frac{\lambda_s}{\lambda_t}\frac{s_{e,s}}{s_{e,t}}ds\right]. \tag{32}$$

To obtain an economic interpretation of the quantity $p^F_{e,t}$, I note that under full liberalization, markets are complete, and so λ_t becomes a constant. Therefore, $p^F_{e,t}$ is the emerging market's price-dividend ratio under full liberalization.

To characterize $h_{e,t}$, note that when the cross-region correlation in output growth, ρ, is one, investing in foreign stock markets no longer offers any diversification benefits, and so even under partial liberalization, markets are complete. Hence, if one defines, $\epsilon = \sqrt{\frac{1-\rho}{2}}$, then $h_{e,t} = p^F_{e,t} + o(\epsilon)$ and one can write:

$$h_{e,t} = p^F_{e,t} + \psi_{e,t}, \tag{33}$$

where

$$\psi_{e,t} = \epsilon h_{e,1,t} + \epsilon^2 h_{e,2,t} + \dots. \tag{34}$$

Together with (30), (33) implies that the partial liberalization emerging market's stock price, $S^P_{e,t}$, is given in terms of the full liberalization price, $S^F_{e,t} = p^F_{e,t}Y_{e,t}$, as shown in the subsequent proposition.

Proposition 3 *The emerging market's stock price under partial liberalization, $S^P_{e,t}$, is given by*

$$S^P_{e,t} = S^F_{e,t} + \nu_{e,t}\psi_{e,t}Y_{e,t}, \tag{35}$$

where $S^F_{e,t}$ is the emerging market's stock price under full liberalization. $\psi_{e,t}$ is an adjustment term reflecting the change in the emerging market's price-dividend ratio upon moving from partial to full liberalization and is defined formally in (A149) in the Appendix.

Equation (35) says that under partial liberalization, the emerging market's stock price is the full liberalization price plus an adjustment term. When Agent o consumes all of world output, i.e. $\nu_{o,t} = 1$, the constrained agent, Agent e, does not affect prices. Consequently, assets can be priced as if markets were complete and the emerging market's stock price under partial liberalization equals the full liberalization price. When Agent o does not consume all of world output, market incompleteness matters and the emerging market's stock price under partial liberalization does not equal the price under full liberalization. To give an economic interpretation to $\psi_{e,t}$, note that (33) implies $\psi_{e,t} = h_{e,t} - p^F_{e,t}$ and (30) implies $S^P_{e,t} = Y_{e,t} h_{e,t}$ when $\nu_{e,t} = 1$. Thus, $h_{e,t}$ is the emerging market's price-dividend ratio when emerging market residents consume all of world output and $\psi_{e,t}$ is the difference between this quantity and the price-dividend ratio under full liberalization. In the Proof of Proposition 3, I give an expression for $p^F_{e,t}$ and show how to derive the higher order terms (see (34)) in the perturbation series expansion for $h_{e,t}$.

Because markets are complete and agents are homogeneous under full liberalization, $p^F_{e,t}$, is a function of a single exogenous state variable, $s_{e,t}$, the share of world output produced in the emerging market. Cochrane, Longstaff, and Santa-Clara (2007) derive a closed-form expression for $f^F_{e,t} = s_{e,t} p^F_{e,t}$, which is the ratio of the stock price to world output, $\frac{S^F_{e,t}}{Y_t}$.

Under partial liberalization, Agent e cannot invest in foreign stock markets, whereas Agent o faces no constraints. That renders both markets incomplete and agents heterogeneous. Consequently $h_{e,t}$ is a function of not only $s_{e,t}$, but also the cross-sectional consumption distribution, described by $\nu_{e,t}$, which is an endogenous state variable. Because of this endogenous state variable deriving stock prices under partial liberalization is harder than in the full liberalization case covered by the results in Cochrane, Longstaff, and Santa-Clara (2007).

Solving for stock prices is also more difficult than in Basak and Cuoco (1998). First, the model considered in this paper has two dividend trees, whereas Basak and Cuoco (1998) have a one dividend tree model. Second, the form of market incompleteness differs. In Basak and Cuoco (1998), the constrained agent can invest only in the risk-free bond, which makes her optimal share of total consumption (the endogenous state variable), independent of the stock price. Therefore, Basak and Cuoco (1998) can pin down the dynamics of the cross-sectional distribution of consumption before solving for the stock price. But in this paper, under partial liberalization the consumption share, $\nu_{e,t}$, *does* depend on stock prices, because the constrained agent, Agent e, can invest in her domestic stock market. The consumption share, $\nu_{e,t}$, thus depends on the emerging market's price-dividend ratio, $p^P_{e,t}$, and hence $g_{e,t}$. One must therefore solve for $\nu_{e,t}$ and $g_{e,t}$ jointly, in contrast with Basak and Cuoco (1998).

Nevertheless I can still solve for stock prices in closed-form, albeit approximately. I use the 'method of matched asymptotic expansions' to obtain a perturbation expansion for $g_{e,t}$ and hence $h_{e,t}$ in (34).[32] Full details are given in the Proof of Proposition 3.

From the closed-form expressions for the emerging market's stock price under partial and full liberalization. I derive the following proposition, which gives the main result of this paper.

Proposition 4 *Under partial liberalization the cost of capital in the emerging market, $\mu^P_{e,t}$, is less than the cost of capital under full liberalization, $\mu^F_{e,t}$, i.e.*

$$\mu^P_{e,t} < \mu^F_{e,t}. \tag{36}$$

To understand why moving from partial to full liberalization increases the emerging market's risk premium, recall that the emerging market's risk premium is higher when the discount rate for the emerging market is more countercyclical (with respect to the world economy). In the subsequent paragraph, I explain how moving from partial to full liberalization impacts the risk-free rate and hence the emerging market's discount rate, $k_{e,t}$, defined in (20).

Under partial liberalization, emerging market residents (Agent e) cannot invest in foreign stock markets, whereas all other investors (Agent o) are unconstrained. Therefore, markets are incomplete, agents are heterogeneous, and so the cross-sectional consumption distribution affects asset prices. A positive shock to world output growth raises the consumption of both agents, but their heterogeneity implies that some will enjoy a greater increase in consumption than others. It then follows from market clearing that one of the two agents will consume a smaller proportion of world output. The two regional stocks are the only claims in the economy which entitle their holders to shares of world output. Emerging market

[32]This approach is often used in applied mathematics to obtain approximate closed-form solutions to differential equations. Textbook treatments are contained in Chapter 5 of Hinch (1991) and Chapters 2 and 3 Kevorkian and Cole (1996). The example on p.52-58 of Hinch (1991) is particularly instructive for someone who is new to this method.

residents can only hold one of these claims, whereas non-residents can hold both. Therefore, when there is a positive shock to world output growth, the share of world output consumed by emerging market residents will fall. Equilibrium asset prices are then less severely effected by the constraint faced by emerging market residents, which reduces the impact of market incompleteness. In other words, overall risk sharing in the economy improves when the constrained agents are squeezed out via a reduction in their share of aggregate consumption. There is then less demand for precautionary savings, which raises the risk-free rate as world output rises. In short, the risk-free rate contains a *procyclical* component, which is present because of the impact of the cross-sectional distribution of consumption.

However, under full liberalization all agents are unconstrained, which makes markets complete and agents homogeneous. The cross-sectional consumption distribution is then irrelevant for asset prices and the procyclical component of the risk-free rate vanishes. The emerging market discount rate is thus more countercyclical, which implies that the discount-rate beta and hence the emerging market's risk premium are higher.

IV Conclusion

Intuition may suggest that more open financial markets lead to better international risk sharing, which decreases the risk premium and hence the cost of capital. The decrease in the cost of capital is beneficial, because it leads to more investment. However, in this paper I show that in a dynamic, general equilibrium economy with full information and rational agents, moving from partial to full liberalization *increases* both the risk premium and cost of capital in an emerging market. Since it is well known that full liberalization lowers the cost of capital relative to autarky, this novel result implies that partial liberalization lowers it *more*.[33] This provides some justification for the limited financial liberalization policies pursued by India and China.

[33] The result that moving from autarky to full liberalization lowers the cost of capital in the emerging market requires the assumption that less than half of world output is produced in the emerging market, i.e. $s_{e,t} < 1/2$. To see why note that under autarky, $\mu_e = \beta + \mu_Y$, whereas under full liberalization, $\mu_e = \beta + \mu_Y + s_e(2s_e - 1)(1 - \rho)\sigma_Y^2 + O(\epsilon^4)$. The assumption $s_{e,t} < 1/2$ is not needed for Propositions 2 and 4, because the constraint on emerging market residents already makes the two regions asymmetric.

The emerging market's cost of capital is lower under partial liberalization than full liberalization, because both the risk-free rate and the emerging market's risk premium are lower under full liberalization. The risk-free rate is lower under full liberalization, because of better risk sharing which leads to less demand for precautionary savings. To understand why the emerging market's risk premium falls, I split the emerging market's beta into a cash-flow and discount-rate beta. I then show that moving from partial to full liberalization leads to more countercyclicality in the emerging market's discount rate which increases its discount-rate beta. That increases the risk premium and cost of capital. My result and the underlying explanation suggest avenues for both empirical and theoretical research.

On the empirical side, studying the relative effects of different degrees of liberalization on the risk premium and cost of capital could be important. To the best of my knowledge, the existing empirical literature does not do this. Furthermore, analyzing the impact of liberalization on cash-flow and discount-rate betas separately would allow empiricists to disentangle the real and financial effects of liberalization.

This article studies an exchange economy, which makes it impossible to analyze the effects of a lower cost of capital on investment in new projects and hence growth. Extending the analysis to a production economy where new projects arrive exogenously would overcome this and offers an interesting opportunity for further theoretical research.[34]

[34] One could start with the model in Gomes, Kogan, and Zhang (2003) and alter it to deal with different levels of international risk sharing.

A Proofs

In all proofs, I omit time subscripts wherever possible.

Vector Notation

Since the model assumes there are two dividend trees driven by two different underlying Brownian motions, derivations are made easier by using vector notation rather than the more economically intuitive scalar notation employed in the text. I define the 2-dimensional Brownian motion $\underline{Z} = (Z_1, Z_2)^T$, via

$$Z_i = \begin{cases} \frac{Z_{Y,e}+Z_{Y,o}}{\sqrt{(2(1+\rho))}}, & i = 1. \\ \frac{Z_{Y,e}-Z_{Y,o}}{\sqrt{(2(1-\rho))}}, & i = 2. \end{cases} \tag{A1}$$

$\underline{Z} = (Z_1, Z_2)^T$ is the coordinate mapping process on the canonical probability space $(\Omega, \mathcal{F}, \mathbb{P})$.[35] To rewrite (3) in terms of $\underline{Z}$, I solve (A1) for $Z_{Y,e}$ and $Z_{Y,o}$. Then I substitute the solution into (3). Simplifying the resulting expression gives

$$\frac{dY_i}{Y_i} = \mu_Y dt + \underline{\sigma}_{Y,i}^T d\underline{Z},\ i \in \{o, e\}, \tag{A2}$$

where

$$\underline{\sigma}_{Y,i} = \begin{cases} \left(\sqrt{\frac{1+\rho}{2}}, \sqrt{\frac{1-\rho}{2}}\right)^T \sigma_Y, & i = o. \\ \left(\sqrt{\frac{1+\rho}{2}}, -\sqrt{\frac{1-\rho}{2}}\right)^T \sigma_Y, & i = e. \end{cases} \tag{A3}$$

The vectors $\underline{\sigma}_{Y,i}$, $i \in \{o, e\}$ are written with respect to a particular basis, where the basis vectors $(1, 0)^T$ and $(0, 1)^T$ are proportional to the Brownian vectors $(Z_1, 0)^T$ and $(0, Z_2)^T$, respectively, I refer to this particular basis as the *output basis*.[36]

Sometimes it will be more convenient to work with a different basis, where $(1, 0)^T$ and $(0, 1)^T$ are proportional to the Brownian vectors $(\hat{Z}_1, 0)^T$ and $(0, \hat{Z}_2)^T$, respectively, where

$$\hat{Z}_i = \begin{cases} \frac{Z_{R,e}+Z_{R,o}}{\sqrt{(2(1+\rho_{eo}))}}, & i = 1. \\ \frac{Z_{R,e}-Z_{R,o}}{\sqrt{(2(1-\rho_{eo}))}}, & i = 2. \end{cases} \tag{A4}$$

I refer to this basis as the *market basis*, because $Z_{R,i}$, $i \in \{o, e\}$, are the Brownian motions which appear in the expressions for regional stock market returns in (4). Because the

[35] See p.27-28 of Karatzas and Shreve (1998) for technical details.

[36] See Skiadas (2007) for a rigorous but accessible treatment of dynamic martingale bases.

Brownian motions $Z_{R,i}$, $i \in \{o, e\}$, and the correlation ρ_{eo} are determined endogenously in equilibrium, they will in general be different from $Z_{Y,i}$, $i \in \{o, e\}$, and the correlation ρ. Hence, the market basis is not the same as the output basis.

With respect to the market basis, (4) becomes

$$dR_i = \mu_i dt + \underline{\sigma}_i^T d\underline{\hat{Z}}, \tag{A5}$$

where $\underline{\hat{Z}} = (\hat{Z}_1, \hat{Z}_2)^T$, and

$$\underline{\sigma}_i = \begin{cases} \left(\sqrt{\frac{1+\rho_{eo}}{2}}, \sqrt{\frac{1-\rho_{eo}}{2}}\right)^T \sigma_o, & i = o. \\ \left(\sqrt{\frac{1+\rho_{eo}}{2}}, -\sqrt{\frac{1-\rho_{eo}}{2}}\right)^T \sigma_e, & i = e. \end{cases} \tag{A6}$$

By stacking the row-vectors $\underline{\sigma}_o^T$ and $\underline{\sigma}_e^T$, I obtain the volatility matrix, σ, with respect to the market basis:

$$\sigma = \begin{pmatrix} \sigma_o\sqrt{\frac{1+\rho_{eo}}{2}} & \sigma_o\sqrt{\frac{1-\rho_{eo}}{2}} \\ \sigma_e\sqrt{\frac{1+\rho_{eo}}{2}} & -\sigma_e\sqrt{\frac{1-\rho_{eo}}{2}} \end{pmatrix}. \tag{A7}$$

To demonstrate why using the market basis can simplify calculations, I now obtain the volatility matrix, σ, with respect to the output basis. Rewriting (4) in vector form with respect to the output basis gives

$$dR_i = \mu_i dt + \underline{\sigma}_i^T d\underline{Z}, \tag{A8}$$

where $\underline{\sigma}_i = (\sigma_{i1}, \sigma_{i2})^T$, $i \in \{o, e\}$, and σ_{ij}, $i \in \{o, e\}$, $j \in \{1, 2\}$ must be determined endogenously in equilibrium. The volatility of stock-market returns in Region i is given by

$$\sigma_i = \sqrt{\underline{\sigma}_i^T \underline{\sigma}_i}, \tag{A9}$$

and the cross-country correlation in stock-market returns is

$$\rho_{eo} = \frac{\underline{\sigma}_o^T \underline{\sigma}_e}{\sigma_o \sigma_e}, \tag{A10}$$

both of which are invariant under a change of basis. Therefore, given σ_e, σ_o and ρ_{eo}, one must determine the 4 quantities σ_{ij}, $i \in \{o, e\}$, $j \in \{1, 2\}$ from the 3 equations

$$\sigma_{i1}^2 + \sigma_{i2}^2 = \sigma_i^2, i \in \{o, e\}, \tag{A11}$$

and

$$\sigma_{e1}\sigma_{o1} + \sigma_{e2}\sigma_{o2} = \rho_{eo}\sigma_e\sigma_o, \tag{A12}$$

together with an arbitrary assumption such as $\sigma_{12} = k\sigma_{21}$ for some k. Hence, one obtains the volatility matrix, σ, with respect to the output basis:

$$\sigma = \begin{pmatrix} \sigma_{o1} & \sigma_{o2} \\ \sigma_{e1} & \sigma_{e2} \end{pmatrix}. \tag{A13}$$

Clearly, it is simpler to compute σ under the market basis, as shown in (A7).

Derivation of the Agent-Specific Market Prices of Risk

I now derive Equations (9) and (11) of Section I.B.

Under partial liberalization markets are incomplete, because Agent e cannot invest in the stock market of Region o, whereas Agent o is unconstrained. Hence, market prices of risk are agent specific. To derive these market prices of risk, I work in a fictitious market, which the constrained agent, Agent e, regards as complete (see Cvitanic and Karatzas (1992)). In the fictitious market, the expected return on the stock market of Region o, which Agent e cannot trade in, is adjusted by an amount δ, i.e.

$$\mu_o(\delta) = \mu_o + \delta, \tag{A14}$$

where $\mu_o(\delta)$ is the adjusted return and μ_o is the unadjusted return. The adjustment, δ, is chosen such that in the fictitious market, Agent e does not want to trade in the stock market of Region o. Cvitanic and Karatzas (1992) show that the smallest such δ also makes Agent e as worse off as possible (subject to not wanting to trade in Region o's stock market) and ensures that her portfolio choice in the fictitiously completed market equals her portfolio choice in the incomplete market. For the case of logarithmic preferences that implies the adjustment, δ, is given by

$$\delta = \text{argmin}_\delta[(\mu_o(\delta) - r, \mu_e - r)\mathbf{\Sigma}^{-1}(\mu_o(\delta) - r, \mu_e - r)^T]^{1/2}, \tag{A15}$$

where

$$\mathbf{\Sigma} = \sigma\sigma^T \tag{A16}$$

is the variance-covariance matrix of stock returns.[37] See Karatzas and Shreve (1998) for a rigorous derivation of (A15). Substituting the adjustment, δ, obtained by solving (A15) into the market price of risk vector for Agent e,

$$\underline{\theta}_e = \sigma^{-1}(\mu_o(\delta) - r, \mu_e - r)^T, \tag{A17}$$

[37] Observe that because the bases we have defined depend on Brownian motions, vectors of conditional first moments, such as $(\mu_o(\delta) - r, \mu_e - r)^T$, are the same under each basis.

and working under the market basis gives

$$\underline{\theta}_e = \phi_e \left(\sqrt{\frac{1+\rho_{eo}}{2}}, -\sqrt{\frac{1-\rho_{eo}}{2}} \right)^T, \tag{A18}$$

after some simple algebra.[38] Consequently

$$\theta_e = \phi_e. \tag{A19}$$

It follows from (A7), that under the market basis

$$\underline{\sigma}_e = \sigma_e \left(\sqrt{\frac{1+\rho_{eo}}{2}}, -\sqrt{\frac{1-\rho_{eo}}{2}} \right). \tag{A20}$$

Using the above expression, I rewrite (A18) as

$$\underline{\theta}_e = \phi_e \sigma_e^{-1} \underline{\sigma}_e^T, \tag{A21}$$

which is the basis-independent representation. Working under the output basis, (A17) simplifies to

$$\underline{\theta}_e = \frac{1}{\sqrt{1-\rho_{eo}^2}} \left[\frac{(-\sigma_{e2}, \sigma_{e1})^T}{\sigma_e} \rho_{eo} - \frac{(-\sigma_{o2}, \sigma_{o1})^T}{\sigma_o} \right] \phi_e. \tag{A22}$$

Agent o faces no constraints, so her market price of risk vector, $\underline{\theta}_o$, is given by

$$\underline{\theta}_o = \sigma^{-1} (\mu_o - r, \mu_e - r)^T, \tag{A23}$$

which simplifies to give

$$\underline{\theta}_o = (1-\rho_{eo}^2)^{-1/2} \left\{ \left(\frac{-\sigma_{e2}}{\sigma_e}, \frac{\sigma_{e1}}{\sigma_e} \right) \phi_o - \left(\frac{-\sigma_{o2}}{\sigma_o}, \frac{\sigma_{o1}}{\sigma_o} \right) \phi_e \right\} \tag{A24}$$

and

$$\underline{\theta}_o = \left(\frac{\phi_e + \phi_o}{\sqrt{2(1+\rho_{eo})}}, \frac{\phi_o - \phi_e}{\sqrt{2(1-\rho_{eo})}} \right)^T, \tag{A25}$$

with respect to the output and market bases, respectively.

It follows immediately from the market-basis expression, (A25), that the size of the market price of risk faced by Agent e is given in (11).

Under full liberalization, agents from both countries can invest freely in both stock markets. Consequently, for both agents the market price of risk is given by the right-hand side of (A23).

[38] Recall that ϕ_e is the Sharpe ratio for the emerging market country's stock market and is defined in (10).

Derivation of Equation (16)

Because both agents are logarithmic with the same rate of time preference, β, the following standard result

$$C_i = \beta X_i,\ i \in \{o, e\}, \tag{A26}$$

follows from the first-order condition of the Hamilton-Jacobi-Bellman equation, regardless of any portfolio constraints. Therefore,

$$\frac{C_o + C_e}{\beta} = X_o + X_e. \tag{A27}$$

Market clearing for the consumption good implies $C_o + C_e = Y$ and bond market clearing implies $X_o + X_e = S_W$. It then follows from Equation (A27) that (16) holds.

Proof of Lemma 1

Agent i optimally choses to invest the proportion π_{ij} of her wealth in Region j's stock market. Agent o has logarithmic utility and she is unconstrained under partial liberalization. Therefore, her optimal portfolio under partial liberalization is given by the mean-variance portfolio

$$(\pi_{oo}, \pi_{oe})^T = \mathbf{\Sigma}^{-1} (\mu_o - r, \mu_e - r)^T. \tag{A28}$$

Agent e has logarithmic utility and in the fictitiously completed market she is unconstrained. Therefore, in the fictitiously completed market, Agent e's optimal portfolio is the mean-variance portfolio

$$(\pi_{eo}, \pi_{ee})^T \quad = \quad \mathbf{\Sigma}^{-1} (\mu_o + \delta - r, \mu_e - r)^T. \tag{A29}$$

Cvitanic and Karatzas (1992) show that the optimal portfolio in the fictitiously completed market coincides with the optimal portfolio in the incomplete market. Therefore, (A29) is Agent e's optimal portfolio under partial liberalization.

Equations (A28) and (A29) simplify to give

$$\pi_{oo} = \frac{1}{1-\rho_{eo}^2}\left(\frac{\mu_o - r}{\sigma_o^2} - \beta_{eo}\frac{\mu_e - r}{\sigma_e^2}\right) \tag{A30}$$

$$\pi_{oe} = \frac{1}{1-\rho_{eo}^2}\left(\frac{\mu_e - r}{\sigma_e^2} - \beta_{oe}\frac{\mu_o - r}{\sigma_o^2}\right), \tag{A31}$$

and

$$\pi_{eo} = 0, \tag{A32}$$

$$\pi_{ee} = \frac{\mu_e - r}{\sigma_e^2}, \tag{A33}$$

where $\beta_{eo} = \frac{\rho_{eo}\sigma_e}{\sigma_o}$ and $\beta_{oe} = \frac{\rho_{eo}\sigma_o}{\sigma_e}$.[39]

The stock market clearing conditions are

$$\pi_{oo}\nu_o + \pi_{eo}\nu_e = \eta_o, \tag{A34}$$

$$\pi_{oe}\nu_o + \pi_{ee}\nu_e = \eta_e, \tag{A35}$$

where $\eta_o = S_o/S_W$ and $\eta_e = 1 - \eta_o$. Substituting (A30), (A31), A32) and (A33) into the stock market clearing conditions and solving simultaneously for the regional stock market risk premia, $\mu_o - r$ and $\mu_e - r$ gives

$$\mu_e - r = \eta_e\sigma_e^2 + \eta_o\rho_{eo}\sigma_e\sigma_o, \tag{A36}$$

and

$$\mu_o - r = \eta_o\left[1 + (1 - \rho_{eo}^2)\frac{\nu_e}{\nu_o}\right]\sigma_o^2 + \eta_e\rho_{eo}\sigma_e\sigma_o. \tag{A37}$$

Markets are complete and agents are homogeneous under fully liberalized stock markets. Hence, because all agents are logarithmic they hold the same mean-variance portfolio

$$(\pi_{io}, \pi_{ie})^T = \mathbf{\Sigma}^{-1}(\mu_o - r, \mu_e - r)^T, \ i \in \{o, e\}. \tag{A38}$$

Substituting the above expressions for the optimal portfolios into the stock market clearing conditions in (A34) and (A35), and then solving for risk premia gives (A36) and

$$\mu_o - r = \eta_o\sigma_o^2 + \eta_e\rho_{eo}\sigma_e\sigma_o. \tag{A39}$$

Thus, under both partial and full liberalization, the emerging market's risk premium is given by (A36), which can be rewritten as

$$(\mu_{e,t} - r_t)dt = \eta_{e,t}Var_t(dR_{e,t}) + \eta_{o,t}Cov_t(dR_{e,t}, dR_{o,t}). \tag{A40}$$

The return on the world market portfolio of risky assets, dR_W, is the return on holding both regional stock markets, i.e. $dR_W = \eta_e dR_e + \eta_o dR_o$. Hence, the emerging market's risk premium can be written more succinctly as (13).

Proof of Proposition 1

The proof is straightforward and described in the text.

[39] In simplifying (A28) and (A29) it is easier to work under the market basis.

Proof of Proposition 2

The risk-free rate under full liberalization given in (28) is derived in Cochrane, Longstaff, and Santa-Clara (2007).

Under full liberalization λ is a constant, because markets are complete. But under partial liberalization λ is stochastic, because markets are incomplete. To derive the risk-free rate under partial liberalization I must first derive expressions for the evolution of the stochastic weight, λ, and Agent o's consumption share, ν_o.

From (7), it follows that

$$\lambda_t = \frac{\kappa_o \xi_{o,t}/\xi_{o,0}}{\kappa_e \xi_{e,t}/\xi_{e,0}}. \tag{A41}$$

To simplify the above expression note that with respect to the output basis (8) becomes

$$\frac{d\xi_{i,t}}{\xi_{i,t}} = -r_t dt - \underline{\theta}_{i,t}^T d\underline{Z}_t,\ i \in \{e, o\}. \tag{A42}$$

It follows that in scalar notation

$$\frac{d\xi_{i,t}}{\xi_{i,t}} = -r_t dt - \theta_{i,t} dZ_{\xi_i,t}. \tag{A43}$$

where

$$Z_{\xi_i,t} = \theta_{i,t}^{-1} \underline{\theta}_{i,t}^T \underline{Z}_t. \tag{A44}$$

Solving (A42) gives

$$\xi_{i,t} = \xi_{i,0} e^{-\int_0^t r_s ds} e^{-\int_0^t \frac{1}{2}\theta_{i,s}^2 ds - \int_0^t \underline{\theta}_{i,s}^T d\underline{Z}_s}. \tag{A45}$$

Hence, (A41) implies that

$$\lambda_t = \frac{\kappa_o}{\kappa_e} e^{-\int_0^t (\theta_{o,s}^2 - \theta_{e,s}^2) ds - \int_0^t (\underline{\theta}_{o,s} - \underline{\theta}_{e,s})^T d\underline{Z}_s}. \tag{A46}$$

Applying Ito's Lemma gives

$$\frac{d\lambda}{\lambda} = \mu_\lambda dt + \underline{\sigma}_\lambda^T d\underline{Z}, \tag{A47}$$

where

$$\mu_\lambda = \underline{\theta}_e^T (\underline{\theta}_e - \underline{\theta}_o), \tag{A48}$$

$$\underline{\sigma}_\lambda = \underline{\theta}_e - \underline{\theta}_o. \tag{A49}$$

Or in scalar notation,

$$\frac{d\lambda}{\lambda} = \mu_\lambda dt + \sigma_\lambda dZ_\lambda, \tag{A50}$$

where

$$\sigma_\lambda = \sqrt{(\underline{\theta}_e - \underline{\theta}_o)^T(\underline{\theta}_e - \underline{\theta}_o)}, \tag{A51}$$

$$Z_\lambda = \sigma_\lambda^{-1}(\underline{\theta}_e - \underline{\theta}_o)^T \underline{Z}. \tag{A52}$$

When stock markets are partially liberalized, the market basis expressions for $\underline{\theta}_e$ and $\underline{\theta}_o$ are given by (A18) and (A25), respectively. Substituting these expressions into (A48), (A49), (A51) and (A52) gives

$$\mu_\lambda = 0, \tag{A53}$$

$$\underline{\sigma}_\lambda = -\frac{\phi_o - \rho_{eo}\phi_e}{\sqrt{1-\rho_{eo}^2}}\frac{(-\sigma_{e2}, \sigma_{e1})^T}{\sigma_e} = -\frac{(-\sigma_{e2}, \sigma_{e1})^T}{\sigma_e}\eta_o\sqrt{1-\rho_{eo}^2}\frac{\sigma_o}{\nu_o} \tag{A54}$$

$$\sigma_\lambda = \frac{\phi_o - \rho_{eo}\phi_e}{\sqrt{1-\rho_{eo}^2}} = \frac{\eta_o}{\nu_o}\sqrt{1-\rho_{eo}^2}\sigma_o, \tag{A55}$$

$$Z_\lambda = -\frac{(-\sigma_{e2}, \sigma_{e1})}{\sigma_e}\underline{Z}, \tag{A56}$$

after some simple algebra.

One can also work under the market basis to evaluate Z_λ via

$$Z_\lambda = \sigma_\lambda^{-1}(\underline{\theta}_e - \underline{\theta}_o)^T \underline{\hat{Z}}. \tag{A57}$$

Under the market basis, one obtains

$$\underline{\sigma}_\lambda = \underline{\theta}_e - \underline{\theta}_o = -\frac{\phi_o - \rho_{eo}\phi_e}{\sqrt{1-\rho_{eo}^2}}\left(\sqrt{\frac{1-\rho_{eo}}{2}}, \sqrt{\frac{1+\rho_{eo}}{2}}\right)^T. \tag{A58}$$

Hence

$$Z_\lambda = -\left(\sqrt{\frac{1-\rho_{eo}}{2}}, \sqrt{\frac{1+\rho_{eo}}{2}}\right)^T \underline{\hat{Z}}, \tag{A59}$$

under the market basis.

Applying Ito's Lemma to $\nu_o = \frac{1}{1+\lambda}$ and recalling that $\mu_\lambda = 0$ yields

$$d\nu_o = \mu_{\nu_o}dt + \underline{\sigma}_{\nu_o}^T d\underline{Z}, \tag{A60}$$

where

$$\mu_{\nu_o} = \nu_o\nu_e^2\sigma_\lambda^2, \tag{A61}$$

$$\underline{\sigma}_{\nu_o} = -\nu_o\nu_e\underline{\sigma}_\lambda. \tag{A62}$$

In scalar notation

$$d\nu_o = \mu_{\nu_o} dt + \sigma_{\nu_o} dZ_\lambda, \tag{A63}$$

where

$$\mu_{\nu_o} = \nu_o \nu_e^2 \sigma_\lambda^2, \tag{A64}$$

$$\sigma_{\nu_o} = -\nu_o \nu_e \sigma_\lambda. \tag{A65}$$

To obtain (27), the risk-free rate under partial liberalization, I apply Ito's Lemma to (26) and equate drift terms with those in (8). Applying Ito's Lemma to (26) and simplifying gives

$$\begin{aligned} \frac{d\xi}{\xi} &= -\left[\beta + \mu_Y - (1 - 2(1-\rho)s_e s_o)\sigma_Y^2 - \frac{\underline{\sigma}_{\nu_o}^T (s_e \underline{\sigma}_{Y,e} + s_o \underline{\sigma}_{Y,o})}{\nu_o}\right] dt \\ &- \left[(s_e \underline{\sigma}_{Y,e} + s_o \underline{\sigma}_{Y,o}) + \frac{\underline{\sigma}_{\nu_o}}{\nu_o}\right]^T d\underline{Z}. \end{aligned} \tag{A66}$$

Therefore, the risk-free rate under partial liberalization, r_P, is given by

$$r_P = r_F - \frac{\underline{\sigma}_{\nu_o}^T (s_e \underline{\sigma}_{Y,e} + s_o \underline{\sigma}_{Y,o})}{\nu_o}, \tag{A67}$$

where r_F is the risk-free rate under full liberalization given by (28). By defining $\rho_{\nu_o,Y}$ as the correlation between shocks to percentage changes in ν_o and shocks to world output growth, (A67) can be rewritten as (27).

Proof of Proposition 3

To characterize the stock price in Region o under partial liberalization, I substitute (26) into

$$S_{o,t}^P = E_t \left[\int_t^\infty \frac{\xi_s}{\xi_t} Y_{o,s} dt\right] \tag{A68}$$

to obtain

$$S_{o,t}^P = E_t \left[\int_t^\infty e^{-\beta(s-t)} \left(\frac{C_{o,s}}{C_{o,t}}\right)^{-1} Y_{o,s} dt\right]. \tag{A69}$$

It follows that

$$S_{o,t}^P = Y_t f_{o,t}^P, \tag{A70}$$

where

$$f_{o,t}^P = E_t \left[\int_t^\infty e^{-\beta(s-t)} \frac{1+\lambda_s}{1+\lambda_t} s_{o,s} ds\right]. \tag{A71}$$

Simplifying the above equation gives

$$
\begin{aligned}
f^P_{o,t} &= E_t\left[\int_t^\infty e^{-\beta(s-t)}\left(\frac{1}{1+\lambda_t}+\frac{\lambda_t}{1+\lambda_t}\frac{\lambda_s}{\lambda_t}\right)s_{o,s}ds\right] && \text{(A72)}\\
&= \frac{1}{1+\lambda_t}E_t\left[\int_t^\infty e^{-\beta(s-t)}s_{o,s}\right]ds \\
&+ \frac{\lambda_t}{1+\lambda_t}E_t\left[\int_t^\infty e^{-\beta(s-t)}\frac{\lambda_s}{\lambda_t}s_{o,s}ds\right] && \text{(A73)}
\end{aligned}
$$

Therefore

$$f^P_{o,t} = \frac{1}{1+\lambda_t}\left(f^F_{o,t}+\lambda_t g_{o,t}\right), \tag{A74}$$

where

$$f^F_{o,t} = E_t\int_t^\infty e^{-\beta(s-t)}s_{o,s}ds, \tag{A75}$$

and

$$g_{o,t} = E_t\int_t^\infty e^{-\beta(s-t)}\frac{\lambda_s}{\lambda_t}s_{o,s}ds. \tag{A76}$$

Hence,

$$f^P_{o,t} = \nu_{o,t}f^F_{o,t} + \nu_{e,t}g_{o,t}. \tag{A77}$$

Under partial liberalization the price-dividend ratio in Region o is $p^P_{o,t} = s_{o,t}f^P_{o,t}$. The price-dividend ratio in Region o under full liberalization is $p^F_{o,t} = s_{o,t}f^F_{o,t}$. Thus, by defining $h_{o,t} = s_{o,t}g_{o,t}$ it follows from (A77) that

$$\frac{S^P_{o,t}}{Y_t} = p^P_{o,t} = \nu_{o,t}p^F_{o,t} + \nu_{e,t}h_{o,t}. \tag{A78}$$

Similarly one can derive (30).

Under full liberalization, markets would be complete and hence λ would be a constant. Therefore, under full liberalization $\frac{S^F_{o,t}}{Y_t} = f^F_{o,t}$. To evaluate $f^F_{o,t}$, I define the new variable $x_t = \ln\frac{s_{o,t}}{s_{e,t}}$. From Ito's Lemma it follows that

$$ds_o = \mu_s dt + \underline{\sigma}_s^T d\underline{Z}, \tag{A79}$$

where

$$
\begin{aligned}
\mu_s &= s_o s_e(s_e - s_o)(1-\rho)\sigma_Y^2, && \text{(A80)}\\
\underline{\sigma}_s &= s_o s_e(\underline{\sigma}_{Y,o} - \underline{\sigma}_{Y,e}). && \text{(A81)}
\end{aligned}
$$

Hence

$$dx = \underline{\sigma}_x^T d\underline{Z}, \tag{A82}$$

where

$$\underline{\sigma}_x = \underline{\sigma}_{Y,o} - \underline{\sigma}_{Y,e}. \tag{A83}$$

Therefore,

$$dx = \sigma_x dZ_x, \tag{A84}$$

where

$$\sigma_x = \sqrt{2(1-\rho)}\sigma_Y, \tag{A85}$$

and

$$Z_x = [2(1-\rho)]^{-1/2}\sigma_Y^{-1}(\underline{\sigma}_{Y,o} - \underline{\sigma}_{Y,e})\underline{Z}. \tag{A86}$$

The Feynman-Kac Theorem then implies

$$\frac{1}{2}\sigma_x^2 \frac{d^2 f_o^F}{dx^2} - \beta f_o^F + \frac{1}{1+e^{-x}} = 0. \tag{A87}$$

To find the boundary conditions for the above inhomogeneous linear ordinary differential equation (ode), observe that (A79) implies that $ds_o|_{s_o=0} = ds_o|_{s_o=1} = 0$. Therefore $s_o = 0$ and $s_o = 1$ are absorbing boundaries. Hence, when $s_o = 0$, $S_o = 0$, implying that $f_o^F = 0$. When $s_o = 1$, $S_o^F = \frac{Y}{\beta} = \frac{Y_o}{\beta}$ and hence $f_o^F = \frac{1}{\beta}$. Consequently, I obtain the boundary conditions

$$\lim_{x\to-\infty} f_o^F = 0 \text{ and } \lim_{x\to\infty} f_o^F = \frac{1}{\beta}. \tag{A88}$$

From Cochrane, Longstaff, and Santa-Clara (2007), I know that the solution of (A87) subject to the above boundary conditions is

$$f_o^F = \frac{1}{2\beta}\left(H(1,a,1+a,-e^{-x}) + \frac{a}{1+a}e^x H(1,1+a,2+a,-e^x)\right), \tag{A89}$$

where

$$a = \sqrt{\frac{\beta}{(1-\rho)\sigma_Y^2}}, \tag{A90}$$

and $H(\alpha_1, \alpha_2, \alpha_3, z)$ is the hypergeometric function, which has the integral representation

$$H(\alpha_1,\alpha_2,\alpha_3,z) = \frac{\Gamma(\alpha_3)}{\Gamma(\alpha_2)\Gamma(\alpha_3-\alpha_2)}\int_0^1 t^{\alpha_2-1}(1-t)^{\alpha_3-\alpha_2-1}(1-tz)^{-\alpha_1}dt, \tag{A91}$$

$$Re(\alpha_3) > Re(\alpha_2) > 0.$$

I now reduce the problem of solving the inhomogeneous linear ode (A87) to the simpler problem of solving a series of linear algebraic equations. That enables me to find an approximate, but highly accurate closed-form expression for f_o^F, which is much simpler than (A89). I start by defining

$$\rho = 1 - 2\epsilon^2. \tag{A92}$$

I use a perturbation expansion of the form

$$f_o^F = f_{o,0}^F + \epsilon f_{o,1}^F + \epsilon^2 f_{o,2}^F + \ldots \tag{A93}$$

to approximate f_o^F. When $\epsilon = 0$, all terms in the linear differential equation (A87) containing derivatives vanish, so (A87) reduces to the linear algebraic equation:

$$-\beta f_{o,0}^F + \frac{1}{1 + e^{-x}} = 0. \tag{A94}$$

Hence,

$$f_{o,0}^F = \frac{1}{\beta(1 + e^{-x})}. \tag{A95}$$

In general, given the terms $f_{o,0}^F, f_{o,1}^F, \ldots, f_{o,n}^F$ in the perturbation expansion of f_o^F, I obtain $f_{o,n+1}^F$ by substituting

$$f_o^F = \sum_{i=0}^{n+1} \epsilon^i f_{o,i}^F \tag{A96}$$

into (A87) and solving the resulting linear algebraic equation in $f_{o,n+1}^F$. In this way, I can show that

$$\begin{aligned}
f_{o,1}^F &= 0, && \text{(A97)}\\
f_{o,2}^F &= \frac{2e^x(1-e^x)}{(1+e^x)^3}\frac{\sigma_Y^2}{\beta^2}, && \text{(A98)}\\
f_{o,3}^F &= 0, && \text{(A99)}\\
f_{o,4}^F &= \frac{4\,e^x\,\left(1 - 11\,e^x + 11\,e^{2\,x} - e^{3\,x}\right)}{\left(1+e^x\right)^5}\frac{\sigma_Y^4}{\beta^3}. && \text{(A100)}
\end{aligned}$$

Therefore,

$$\begin{aligned}
f_o^F &= \frac{s_o}{\beta} - s_o s_e (s_o - s_e)\frac{(1-\rho)\sigma_Y^2}{\beta^2} && \text{(A101)}\\
&- s_e s_o (s_o - s_e)((s_o - s_e)^2 - 8 s_e s_o)\frac{(1-\rho)^2\sigma_Y^4}{\beta^3} + O(\epsilon^6).
\end{aligned}$$

Since an exact closed-form expression is available for f_o^F one can compute the percentage error of the above fourth order approximation for given parameter values. With $\mu_Y = \sigma_Y = 0.03$ per annum and $\beta = 0.01$ per annum, the upper bounds on the size of the percentage error for $\rho = 0.8, 0.2, 0.01, 0, -0.5, -1$ are, $0.0007\%, 0.04\%, 0.065\%, 0.065\%, 0.2\%, 0.5\%$, respectively. For the first and second derivatives of f_o^F with respect to s_o, the percentage errors are ten times as large, which is still small, especially for $\rho \geq 0$. This is important, because I shall use the same approximation method to find a perturbation series expansion for g_o, which is a related but more difficult problem, where all solution methods, whether numerical or closed-form will yield approximate solutions.

I now find an approximate closed-form expression for g_o. Observe that λ is a local martingale. If σ_λ is bounded, then λ is an exponential martingale and I can define a new measure $\mathbb{P}'$ via

$$\mathbb{P}'(A_T) = E_t[1_{A_T}\lambda_T],\ \forall t, T \in [0,\infty),\ t \leq T, \tag{A102}$$

where A_T is an event which occurs at time T and $\mathbb{P}'(A_T)$ is the probability of its occurence based on information known at time t (see p.28-29 of Karatzas and Shreve (1998) for details). Exploiting the change of measure, I rewrite (A76) as

$$g_{o,t} = E_t^{\mathbb{P}'} \int_t^\infty e^{-\beta(s-t)} s_{o,s} ds. \tag{A103}$$

When $\epsilon = 0$, λ is a constant, $\mathbb{P}'$ coincides with original measure $\mathbb{P}$, and thus from (A103) it follows that $g_o = f_o^F$. Therefore,

$$g_o = f_o^F + \Delta_o, \tag{A104}$$

for some function Δ_o, which is $o(\epsilon)$, and depends on x and $\nu_o = \frac{1}{1+\lambda}$. From (A103) one can see that $g_{o,t}$ satisfies an infinite-horizon backward stochastic differential equation (BSDE). The coefficients of the the BSDE will depend on $\nu_{o,t}$, which satisfies a forward stochastic differential equation (FSDE), given in (A60). But the coefficients of the FSDE depend on $g_{o,t}$. Thus the FSDE and the BSDE are coupled together into a forward-backward stochastic differential equation (FBSDE) system. Lejay (2004) shows that the Feynman-Kac Theorem still applies and can be used to show that g_o satisfies the following inhomogeneous elliptic partial differential equation (pde):

$$\mu_x^{\mathbb{P}'} g_{o,x} + \mu_{\nu_o}^{\mathbb{P}'} g_{o,\nu_o} + \frac{1}{2}\sigma_x^2 g_{o,xx} + \sigma_x \sigma_{\nu_o} \rho_{x\nu_o} g_{o,x\nu_o} + \frac{1}{2}\sigma_{\nu_o}^2 g_{o,\nu_o\nu_o} - \beta g_o + \frac{1}{1+e^{-x}} = 0, \tag{A105}$$

where

$$\mu_x^{\mathbb{P}'} = \underline{\sigma}_x^T \underline{\sigma}_\lambda, \tag{A106}$$

$$\mu_{\nu_o}^{\mathbb{P}'} = -\nu_o^2 \nu_e \sigma_\lambda^2. \tag{A107}$$

Equations (A106) and (A107) give the drifts under the measure $\mathbb{P}'$ of x and ν_o, respectively, and are derived using Girsanov's Theorem.

Observe that the inhomogeneous term in (A105) depends purely on x. If I had used the Feynman-Kac Theorem to derive a pde from (A76) , without first changing the measure, the inhomogeneous term in the resulting pde would depend on both ν_o and x.[40]

The boundary conditions required to solve (A105) are given below:

$$\lim_{x \to -\infty} g_o = 0, \tag{A108}$$

$$\lim_{x \to \infty} g_o = \frac{1}{\beta}, \tag{A109}$$

$$\lim_{\nu_o \to 1} \nu_e g_o = 0, \tag{A110}$$

$$\left. \frac{\partial g_o}{\partial \nu_o} \right|_{\nu_o = 0} = 0. \tag{A111}$$

I now derive the above boundary conditions. If $s_o = 0$, then all of world output is produced in Region e. The constraint Agent e faces by not being allowed to invest in Region o's stock market becomes irrelevant. Thus, the economy reduces to one with a single dividend tree and complete markets. Hence,

$$S_o = 0, \tag{A112}$$

$$S_e = \frac{Y_e}{\beta}. \tag{A113}$$

and λ and thus ν_o are constants determined by the initial conditions. Therefore,

$$0 = \left. \frac{S_o}{Y} \right|_{s_o = 0} = 0 = \nu_o f_o^F|_{s_o=0} + \nu_e g_o|_{s_o=0} = \nu_e g_o|_{s_o=0}, \tag{A114}$$

which implies $g_o|_{s_o=0} = 0$ and hence (A108). If $s_o = 1$ all of world output is produced in Region o. The economy reduces to one with a single dividend tree, the claim to which can be traded freely only by Agent o. Both agents can borrow and lend to each other freely at the risk-free rate. This is the economy studied in Basak and Cuoco (1998). I know from Basak and Cuoco (1998) that

$$S_o = \frac{Y_o}{\beta}. \tag{A115}$$

[40] I also derived and solved the pde for g_o based on (A76), for which I did *not* assume σ_λ is bounded. Up to fourth order in ϵ, the resulting solution is identical to the one obtained by solving (A105), which was obtained by assuming σ_λ is bounded.

Therefore,

$$\left.\frac{S_o}{Y}\right|_{s_o=1} = \frac{1}{\beta} = f_o^P|_{s_o=1} = \nu_o f_o^F|_{s_o=1} + \nu_e g_o|_{s_o=1} = \nu_o \frac{1}{\beta} + \nu_e g_o|_{s_o=1}, \tag{A116}$$

which implies $g_o|_{s_o=1} = \frac{1}{\beta}$ and hence (A109). When $\nu_o = 1$, Agent o, who faces no constraints, consumes all world output, so prices are as if markets were complete. Therefore,

$$f_o^P|_{\nu_o=1} = f_o^F, \tag{A117}$$

which is satisfied as long as (A110) holds. As $\nu_o \to 0$, Agent e consumes all output. Because Agent e has a worse investment opportunity set than Agent o, Agent e cannot consume all output for more than an instant. Therefore the boundary at $\nu_o = 0$ must be a reflecting boundary. Hence, (A111) holds. Having derived boundary conditions, I now solve (A105).

Observe that (A105) is an elliptic partial differential equation, where the coefficients depend on g and is partial derivatives in a nonlinear fashion. Therefore (A105) is a *quasilinear* elliptic partial differential equation. Hence (A105) is a considerably more complicated differential equation than (A87). Despite this, I can use the same method as was used in solving in (A87) to reduce the problem of solving (A105) to the problem of solving a series of linear algebraic equations. Solving these linear algebraic equations gives a perturbation expansion in ϵ around 0, which is an approximate expression for g_o.

Because g_o is given by (A104) and I know the relevant perturbation expansion for f_o^F and that $\Delta = o(\epsilon)$, I start by expanding Δ_o in ϵ around 0:

$$\Delta_o = \sum_{i=1}^{n+1} \Delta_{o,i} \epsilon^i. \tag{A118}$$

Note that some coefficients of the pde (A105) depend on $\underline{\sigma}_\lambda$, which in turn depends on the regional stock market volatilities, σ_o, σ_e and the cross-regional stock market return correlation, ρ_{eo}. Applying Ito's Lemma to $S_o^P = Y f_o^P$ gives the following expression for $\underline{\sigma}_o$:

$$\underline{\sigma}_o = \frac{\partial f_o^P}{\partial s_o} \underline{\sigma}_s + \frac{\partial f_o^P}{\partial \nu_o} \underline{\sigma}_{\nu_o} + s_e \underline{\sigma}_{Y,e} + s_o \underline{\sigma}_{Y,o}. \tag{A119}$$

Applying Ito's Lemma to (16) and equating diffusion coefficients gives

$$\underline{\sigma}_e = \frac{\frac{1}{\beta}(s_o \underline{\sigma}_{Y,o} + s_e \underline{\sigma}_{Y,e}) - f_o^P \underline{\sigma}_o}{\frac{1}{\beta} - f_o^P}, \tag{A120}$$

which gives the emerging market's stock return volatility vector in terms of Region o's stock return volatility vector. Thus, one can see that $\underline{\sigma}_o$, $\underline{\sigma}_e$ and hence σ_o, σ_e and ρ_{eo} depend on $\underline{\sigma}_{\nu_o}$ and hence $\underline{\sigma}_\lambda$. Substituting (A54) into (A119) gives

$$\underline{\sigma}_o = \frac{\partial f_o^P}{\partial s_o}\underline{\sigma}_s + \frac{\partial f_o^P}{\partial \nu_o}\nu_e \eta_o \sigma_o \sqrt{1-\rho_{eo}^2}\frac{(-\sigma_{e2}, \sigma_{e1})^T}{\sigma_e} + s_e \underline{\sigma}_{Y,e} + s_o \underline{\sigma}_{Y,o}. \tag{A121}$$

To find a perturbation expansion for $\underline{\sigma}_o$ in terms of exogenous variables together with ν_o and f_o^P, I substitute (A120),

$$f_o^P = f_{o,0}^P + \epsilon f_{o,1}^P + \epsilon^2 f_{o,2}^P + \ldots, \tag{A122}$$

where

$$f_{o,i}^P = f_{o,i}^F + \nu_e \Delta_{o,i}, \tag{A123}$$

and

$$\underline{\sigma}_o = \underline{\sigma}_{o,0} + \epsilon \underline{\sigma}_{o,1} + \epsilon^2 \underline{\sigma}_{o,2} + \ldots, \tag{A124}$$

into (A121) and compare coefficients of ϵ to solve for $\underline{\sigma}_{o,0}$, $\underline{\sigma}_{o,1}$, $\underline{\sigma}_{o,2}$, ... in terms of exogenous variables, ν_o and $f_{o,0}^P$, $f_{o,1}^P$, $f_{o,2}^P$, ... After some algebra, it follows that

$$\sigma_{o1} = \sigma_Y\left(1 - \frac{\epsilon^2}{2}\right) + O(\epsilon^3) \tag{A125}$$

$$\sigma_{o2} = \sigma_Y\left(-\frac{\epsilon}{2} + 2\beta f_{o,1}^P\left(\frac{1}{\nu_o} - \frac{1}{s_o} + \nu_e\frac{\partial \ln f_{o,1}^P}{\partial \ln \nu_o} + \frac{s_e}{s_o}\frac{\partial \ln f_{o1}^P}{\partial \ln s_o}\right)\epsilon^2\right) + O(\epsilon^3) \tag{A126}$$

By substituting (A125) and (A126) into (A120), I derive an expression for $\underline{\sigma}_e$ in terms of exogenous variables, ν_o and f_o^P. Hence, I have expressions for all the elements of the matrix σ in terms of exogenous variables, ν_o and f_o^P. Thus, I can use (A54) to obtain an expression for $\underline{\sigma}_\lambda$ in terms of exogenous variables, ν_o and f_o^P. Using this expression for $\underline{\sigma}_\lambda$ I can write the coefficients of the pde (A105) in terms of exogenous variables, the endogenous state variable ν_o and f_o^P. Having made the dependence of the coefficients of the pde (A105) on exogenous state variables, ν_o and f_o^P explicit, I substitute the expansion (A118) into the pde, simplify and set the coefficients of ϵ, ϵ^2, ... equal to zero. To leading order in ϵ, the terms in the pde (A105) involving derivatives all vanish, giving a linear algebraic equation, which implies that $\Delta_{o,1}$=0. I then obtain the terms $\Delta_{o,2}$, $\Delta_{o,3}$, ..., by solving linear algebraic equations, in the same way I solved for $f^F_{o,0}$, $f^F_{o,1}$, ..., etc. Thus,

$$\Delta_{o,1} = 0, \tag{A127}$$

$$\Delta_{o,2} = -4\frac{s_o^2 s_e}{\nu_o}\frac{\sigma_Y^2}{\beta^2}, \tag{A128}$$

$$\Delta_{o,3} = 0, \tag{A129}$$

implying that

$$g_o = f_o^F - 4\epsilon^2\frac{s_o^2 s_e}{\nu_o}\frac{\sigma_Y^2}{\beta^2} + O(\epsilon^4). \tag{A130}$$

The fourth order term, $\Delta_{o,4}$, is algebraically cumbersome and available upon request. Because (A130) was obtained by solving linear algebraic equations, it contains no constants of integration, making it impossible to satisfy all the boundary conditions. It is straightforward to see that (A111) is the only boundary condition, which is not satisfied. Thus, the approximate solution will apply away from the boundary where $\nu_o = 0$, i.e. when the share of world output consumed by non-emerging market residents is not small. This is the region which is of economic interest when studying the cost of capital in an emerging market, because one can assume that an emerging market's residents do not consume a large fraction of world output. Therefore for the economic analysis in this paper, it not necessary to obtain a solution to (A105), which is valid close to $\nu_o = 0$. The region close to $\nu_o = 0$ is called the *boundary layer.*

But to show that the boundary layer is order one in ϵ, I do need to derive a solution of (A105), which satisfies all 4 boundary conditions. This solution gives an extra term in the perturbation expansion (A130). Compared with existing terms, the extra term is non-negligible very close to the boundary at $\nu_o = 0$ and is negligibly small away from the boundary layer. I can infer the thickness of the boundary layer from this extra *boundary layer term.* I use the 'method of matched asymptotic expansions' derive the boundary layer term. This method is standard in applied mathematics—see Chapter 5 of Hinch (1991) and Chapters 2 and 3 of Kevorkian and Cole (1996). I start by introducing a rescaling with the coordinate

$$\chi = \frac{\nu_o}{\epsilon}. \tag{A131}$$

The partial differential equation (A105) now becomes

$$\mu_x^{\mathbb{P}'} g_{o,x} + \mu_{\nu_o}^{\mathbb{P}'}\frac{1}{\epsilon}g_{o,\chi} + \frac{1}{2}\sigma_x^2 g_{o,xx} + \sigma_x\sigma_{\nu_o}\rho_{x\nu_o}\frac{1}{\epsilon}g_{o,x\chi} + \frac{1}{2}\sigma_{\nu_o}^2\frac{1}{\epsilon^2}g_{o,\chi\chi} - \beta g_o + \frac{1}{1+e^{-x}} = 0. \tag{A132}$$

To leading order, the first and second derivative terms no longer vanish. It follows from (A132) that $\Delta_{o,1}$ satisfies the nonlinear second order ordinary differential equation

$$-\chi(\beta\Delta_{o,1,\chi}-(1-s_o))^2\Delta_{o,1}+\frac{1}{\beta}2(1-s_o)^2 s_o^2\frac{\sigma_Y^2}{\beta}(2\beta\Delta_{o,1,\chi}+\beta\chi\Delta_{o,1,\chi\chi}-2(1-s_o))=0. \quad \text{(A133)}$$

Using the linear (in χ) approximation $\chi(\beta\Delta_{o,1,\chi}-(1-s_o))^2 \approx \chi(A-(1-s_o))^2$, where $A=\beta\Delta_{o,1,\chi}|_{\chi=\overline{\chi}}$ for some constant $\overline{\eta}$, reduces (A133) to the linear second-order ordinary differential equation

$$-(A-(1-s_o))^2\Delta_{o,1}+\frac{1}{\beta}2(1-s_o)^2 s_o^2\frac{\sigma_Y^2}{\beta}(2\beta\Delta_{o,1,\chi}+\beta\chi\Delta_{o,1,\chi\chi}-2(1-s_o))=0, \quad \text{(A134)}$$

which has the general solution

$$\Delta_{o,1}=\frac{1}{\chi}\left[-\frac{4(1-s_o)^3 s_o^2}{(A-(1-s_o))^2}\frac{\sigma_Y^2}{\beta^2}+C_1(s_o)e^{\chi\sqrt{\frac{\beta}{2}}\frac{A-(1-s_o)}{s_o(1-s_o)\sigma_Y}}+C_2(s_o)e^{-\chi\sqrt{\frac{\beta}{2}}\frac{A-(1-s_o)}{s_o(1-s_o)\sigma_Y}}\right], \quad \text{(A135)}$$

where $C_1(s_o)$ and $C_2(s_o)$ are arbitrary functions of s_o. I now have two perturbation expansions for Δ_o. There is the original expansion in (A130), which is called the *outer expansion*, because it is valid outside the boundary layer. And there is the new expansion, which is called the *inner expansion*, because is valid inside the boundary layer. The constant $\overline{\chi}$ and the arbitrary functions $C_1(s_o)$, $C_2(s_o)$, are determined by *matching* the inner and outer expansions, so they are equal in an overlap region where ν_o is small and χ is large, i.e. $\epsilon \ll \nu_o = \epsilon\chi \ll 1$. To carry out the matching, I express both ν_o and χ in terms of an intermediate variable η:

$$\eta=\frac{\nu_o}{\epsilon^\alpha}=\chi\epsilon^{1-\alpha}. \quad \text{(A136)}$$

Therefore, I obtain the following two expressions for the outer and inner expansions, respectively. From (A127), (A128) and (A129)

$$\Delta_o=-\epsilon^{2-\alpha}4s_o(1-s_o)\frac{s_o}{\eta}\frac{\sigma_Y^2}{\beta^2}+O(\epsilon^4), \quad \text{(A137)}$$

and

$$\Delta_o=\frac{\epsilon^{2-\alpha}}{\eta}\left[-\frac{4(1-s_o)^3 s_o^2}{(A-(1-s_o))^2}\frac{\sigma_Y^2}{\beta^2}+C_1(s_o)e^{\frac{\eta}{\epsilon^{1-\alpha}}\sqrt{\frac{\beta}{2}}\frac{A-(1-s_o)}{s_o(1-s_o)\sigma_Y}}+C_2(s_o)e^{-\frac{\eta}{\epsilon^{1-\alpha}}\sqrt{\frac{\beta}{2}}\frac{A-(1-s_o)}{s_o(1-s_o)\sigma_Y}}\right], \quad \text{(A138)}$$

from (A135). If $\overline{\chi}$ is chosen such that $A = 0$, then to ensure the right-hand side of (A138) is finite as $\epsilon \to 0$, I must set $C_2(s_o) = 0$. Hence, (A138) becomes

$$\Delta_o = \frac{\epsilon^{2-\alpha}}{\eta}\left(-4(1-s_o)s_o^2\frac{\sigma_Y^2}{\beta^2} + C_1(s_o)e^{-\frac{\eta}{\epsilon^{1-\alpha}}\sqrt{\frac{\beta}{2}}\frac{1}{s_o\sigma_Y}}\right), \tag{A139}$$

The above expression is identical with (A137) for positive orders of ϵ if $C_1(s_o) = 4(1-s_o)s_o^2\frac{\sigma_Y^2}{\beta^2})(1+\delta(s_o))$ for some $\delta(s_o)$. which is a function of s_0. Hence,

$$\Delta_o = -\frac{\epsilon^{2-\alpha}}{\eta}4(1-s_o)s_o^2\frac{\sigma_Y^2}{\beta^2}\left(1-(1+\delta(s_o))e^{-\frac{\eta}{\epsilon^{1-\alpha}}\sqrt{\frac{\beta}{2}}\frac{1}{s_o\sigma_Y}}\right), \tag{A140}$$

where $(1+\delta(s_o))e^{-\frac{\eta}{\epsilon^{1-\alpha}}\sqrt{\frac{\beta}{2}}\frac{1}{s_o\sigma_Y}}$ is the boundary layer term, which ensures the boundary condition (A111) is satisfied. Expressing (A140) in terms of ν_o gives

$$\Delta_o = -\epsilon^2 4(1-s_o)\frac{s_o^2}{\nu_o}\frac{\sigma_Y^2}{\beta^2}\left(1-(1+\delta(s_o))e^{-\frac{\nu_o}{\epsilon}\sqrt{\frac{\beta}{2}}\frac{1}{s_o\sigma_Y}}\right). \tag{A141}$$

Note that the boundary layer term $(1+\delta(s_o))e^{-\frac{\nu_o}{\epsilon}\sqrt{\frac{\beta}{2}}\frac{1}{s_o\sigma_Y}}$ becomes exponentially small as $\epsilon \to 0$. To satisfy the boundary condition (A111), the following equation must be satisfied

$$\frac{4\,(1-s_o)\,s_o^2\,\epsilon^2\sigma_Y^2\,\delta(s_o)}{\beta^2\,{\nu_o}^2} - \frac{(1-s_o)\,(1+\delta(s_o))}{\beta} = 0 \tag{A142}$$

Solving the above equation for ν_o tells us that within the boundary layer

$$\nu_o = 2\,s_o\,\epsilon\sigma_Y\sqrt{\frac{1}{\beta}\frac{\delta(s_o)}{1+\delta(s_o)}}. \tag{A143}$$

Therefore, the boundary layer is order one in ϵ. It is not necessary to determine $\delta(s_o)$ for the analysis carried out in this paper. But one could determine $\delta(s_0)$ by applying Ito's Lemma to (A143) to derive a stochastic differential equation (sde) for ν_o and substituting f_o^P into the sde for ν_o in (A60) and comparing coefficients.

Since

$$S_o^P = f_o^F + \nu_e\Delta_o Y, \tag{A144}$$

it follows from (16) that

$$S_e^P = f_e^F + \nu_e\Delta_e Y, \tag{A145}$$

where $\Delta_e = -\Delta_o$ and hence

$$\Delta_e = \epsilon^2 4 s_e \frac{s_o^2}{\nu_o} \frac{\sigma_Y^2}{\beta^2} \left(1 - (1 + \delta(s_o)) e^{-\frac{\nu_o}{\epsilon} \sqrt{\frac{\beta}{2}} \frac{1}{s_o \sigma_Y}} \right) + O(\epsilon^4) \tag{A146}$$

and

$$\psi_e = \epsilon^2 4 \frac{s_o^2}{\nu_o} \frac{\sigma_Y^2}{\beta^2} \left(1 - (1 + \delta(s_o)) e^{-\frac{\nu_o}{\epsilon} \sqrt{\frac{\beta}{2}} \frac{1}{s_o \sigma_Y}} \right) + O(\epsilon^4). \tag{A147}$$

Since Region e is an emerging market, I can confine the analysis to the case where Agent e's consumption share of world output is small, i.e. outside the boundary layer. Hence,

$$\Delta_e = \epsilon^2 4 s_e \frac{s_o^2}{\nu_o} \frac{\sigma_Y^2}{\beta^2} + O(\epsilon^4) \tag{A148}$$

and

$$\psi_e = \epsilon^2 4 \frac{s_o^2}{\nu_o} \frac{\sigma_Y^2}{\beta^2} + O(\epsilon^4). \tag{A149}$$

I check the accuracy of the expansion in the same way as one would check the accuracy of a purely numerical method: by substituting the approximate solution back into the pde (A105) and seeing how close the result is to zero. For parameter values $\beta = 0.02$, $\mu_Y = \sigma_Y = 0.01$ (units of per annum) and $\nu_o > 0.1$, substituting the fourth order expansion into the pde gives values less than $10^{-11}, 10^{-10}, 10^{-9}, 10^{-8}, 10^{-7}, 10^{-4}$ for $\rho = 0.98, 0.92, 0.82, 0.68, 0.50, -1$. Purely numerical methods, such as the method of lines (see Schiesser (1991)) produce errors of similar magnitude.

Proof of Proposition 4

First I determine the stochastic differential equation for ν_o, (A60), in closed form outside the boundary layer. I do this by substituting the expressions for σ_{o1} and σ_{o2} in (A125) and (A126) into (A120). Thus I obtain the volatility matrix σ (with respect to the output basis) in terms of exogenous parameters, the endogenous state variable ν_o and f_o^P. Substituting the perturbation expansion for f_o^P into this expression for the volatility matrix gives σ in terms of terms of exogenous parameters and the endogenous state variable ν_o. With this expression for σ, I can use the expression for $\underline{\sigma}_\lambda$ in (A54) to show, after some algebra, that

$$\underline{\sigma}_\lambda = -\left(\epsilon (0,1)^T + \epsilon^2 (1,0)^T \right) 2 s_o \sigma_Y + O(\epsilon^3). \tag{A150}$$

It then follows from (A61) and (A62) that

$$\mu_{\nu_o} = \epsilon^2 4 s_o^2 \left(\frac{\nu_e}{\nu_o} \right)^2 \sigma_Y^2 + O(\epsilon^4), \tag{A151}$$

and

$$\underline{\sigma}_{\nu_o} = \left(\epsilon(0,1)^T + \epsilon^2(1,0)^T\right) 2 s_o \nu_o \nu_e \sigma_Y + O(\epsilon^3), \tag{A152}$$

with respect to the output basis. The change in the emerging market's risk premium upon moving from partial to full liberalization is given by

$$(\mu^F_{e,t} - r^F_t) - (\mu^P_{e,t} - r^P_t) = Cov_t\left(\frac{dp^F_{e,t}}{p^F_{e,t}} - \frac{dp^P_{e,t}}{p^P_{e,t}}, \frac{dY_t}{Y_t}\right). \tag{A153}$$

Now substituting the expansions for f^P_e (outside the boundary layer) and f^F_e into the above expression and simplifying gives

$$(\mu^F_e - r^F) - (\mu^P_e - r^P) = \frac{16 s_o^2 \epsilon^4 \nu_e \left(\nu_o - 3 s_o \nu_o + s_o^2(1+2\nu_o)\right) {\sigma_Y}^4}{\beta {\nu_o}^2} + O(\epsilon^6). \tag{A154}$$

When considered as a quadratic in s_o, the roots of

$$\nu_o - 3 s \nu_o + s_o^2(1+2\nu_o) \tag{A155}$$

are complex if $\nu_o \in [0,1]$. Therefore (A155) does not change sign if $\nu_o \in [0,1]$. It then follows by inspection that (A155) is strictly positive, which implies that to fourth order in ϵ

$$(\mu^F_e - r^F) - (\mu^P_e - r^P) > 0. \tag{A156}$$

Since, $r^F > r^P$, (36) follows.

B Technical Appendix

The Generalization of the Gordon Growth Formula

I assume that the discounted gains process, $G_{e,t} = \xi_t S_{e,t} + \int_0^t \xi_s Y_{e,s} ds$, is a martingale under the natural measure $\mathbb{P}$, so that the standard pricing equation (29) is valid.

In vector notation

$$\xi_s = \xi_t e^{-\int_t^s \left(r_u + \frac{1}{2}\theta_u^2\right) du - \int_t^s \underline{\theta}_u^T d\underline{Z}_u} \tag{B1}$$

and

$$Y_{e,s} = Y_{e,t} e^{-\left(\mu_Y - \frac{1}{2}\sigma_Y^2\right) du + \underline{\sigma}_{Y,e}^T\left(\underline{Z}_s - \underline{Z}_t\right)}. \tag{B2}$$

Therefore, the integrand in (29) can be expressed as

$$\frac{\xi_s Y_{e,s}}{\xi_t Y_{e,t}} = e^{-\int_t^s r_u + \underline{\theta}_u^T \underline{\sigma}_{Y,e} - \mu_Y du} \frac{M_s}{M_t}, \tag{B3}$$

where M_t is the following local exponential martingale

$$M_t = e^{-\int_t^s (\underline{\theta}_u - \underline{\sigma}_{Y,e})^T (\underline{\theta}_u - \underline{\sigma}_{Y,e}) du - \int_t^s (\underline{\theta}_u - \underline{\sigma}_{Y,e})^T d\underline{Z}_u}. \tag{B4}$$

If the market price of risk, θ_t, is bounded, then M_t is an exponential martingale and I can define the equivalent martingale measure $\mathbb{P}^e$ on $(\Omega, \mathcal{F})$ via

$$\mathbb{P}^e(A_T) = E_t[1_{A_T} M_T], \, \forall t, T \in [0, \infty), \, t \leq T, \tag{B5}$$

where A_T is an event which occurs at time T and $\mathbb{P}^e(A_T)$ is the probability of its occurrence based on information known at time t. It follows that

$$S_{e,t} = E_t^{\mathbb{P}^e} \int_t^{\infty} e^{-\int_t^s k_{e,u} - \mu_Y du} ds, \tag{B6}$$

where

$$k_{e,t} = r_t + \underline{\theta}_t^T \underline{\sigma}_{Y,e}. \tag{B7}$$

The above equation can be rewritten as (20).

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