2007

Microeconomics

Lectures of *Dr. M Idrees*

"The main purpose and ultimate of writing these notes is to enable and to give guide line before attending the lectures of Dr M. Idrees. After reading these notes he/ she will be in position to be attentive in the class".

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M. Sc Economics

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Part-I

Microeconomics

Notes

Economics

It is a subject in which we study about “wealth”. Economics deals with the scare resources. Economics is the science of wants.

Economic Problems:

A. Limited resources**.**
B. Unlimited desires.
There are two types (forms) of desires.

1. Economic desire

An economic desire is the economic activity (Which has the market value) with ability to satisfy human needs.

Example: Teacher and his service that has economic value in the shape of salary.

2. Non-Economic desire

Non-Economic desire is one, which does not have market value.

No one can satisfy all his desires, how can we bridge the gap between unlimited desires and limited resources, until unless this gap will bridged then whole world’s economic activity will stop, i.e. Gold coins rain.

Economic Agents:

A. Households
B. Firms
C. Government

- **Households**

They are the basic consuming units, a person who spends some amount for commodity for consumption to satisfy needs of consumption.

- **Firms**

They are the producer of commodity and are basic producing unit, a person who spends some money to produce good and sale goods to households.

- **Government**

Government is regulatory authority which is the link between households and firms, its role is to check the stability in prices and role also depends upon the running economic system in the country.

Microeconomics

Microeconomics is the branch of economics deals with the choices of individual households and individual firms and their behaviors. Microeconomics also deals with decision analysis of households, firms and working of a particular market. Microeconomics is foundation of economics.

Macroeconomics

Macroeconomics is the branch of economics deals with the operations of a entire economy and economy as whole.

Types of Microeconomic Analysis

- **A.** Positive and Normative Analysis
- **B.** Short Period and Long Period
- **C.** Static, Comparative static and Dynamic
- **D.** Partial and General Analysis

- **Positive and Normative Analysis**

Positive economics deals with the question of "What is?" Positive economics are the statements that start with assumptions and derive some conclusion on the basis of actual data. i.e. "a minimum wage law increases youth unemployment is the statement in positive economics because it can be verified with actual data"

Normative economics deals with the question of "What ought to be or should be". It is concluding type of situation based on assumptions such as "consumer maximizes utility" it arguing that in essence we are assuming that consumer should/ought to maximize utility, i.e. "producer ought to maximize profits".

- **Short Period and Long Period**

Short period is one in which full adjustment is not possible, if one factor is fixed then we can say that it is short period, during which consumers and producers have not had enough time to make all the adjustments to the new situation.

Long period is one during which consumers and producers have had enough time to make all the adjustments to a new situation, all factors are variables.

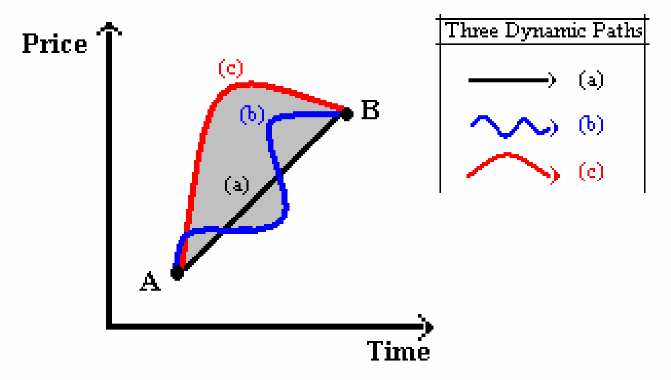
- **Static, Comparative static and Dynamic**

Static is the branch of economics, which deals with the properties of positions of equilibrium in the economic system

$$MC = MR, \qquad \pi = Max$$

Comparative static is the branch of economics in which we discuss by looking at one variable changes and how it changes equilibrium point due to effect of any internal factor on equilibrium (compare equilibrium positions).

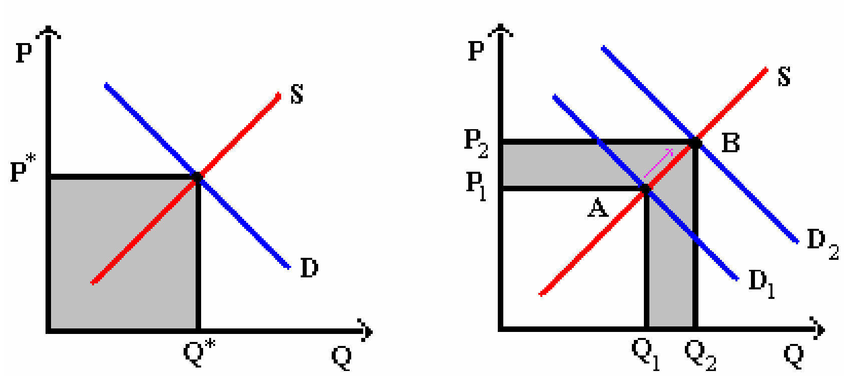
Dynamic is the branch of economics, if there is any external shock then it will be possible to retrain equilibrium position or not, if possible then through which path this entire process discuss in dynamic analysis.



➢ **Partial and General Analysis**

Partial analysis is study of one segment of the society, we change only one variable other things constant. i.e. "A strike which occurs only in one industry with the impact on other industries almost negligible"

General analysis is concerned with the study of the effect of certain changes and after all the interactions in the economy has taken place. i.e. "The import quota on automobiles will have impact on the gasoline, steel, aluminum, glass, platinum and other industries, there is also have further effect on automobile industries".



Frame Work of Microeconomics

There are two sides:

a. Demand side
b. Supply side

Demand side analysis focuses on the behavior of consumer, theory of consumer behavior and finally consumer side. Final out come of theory of consumer is demand.

Supply side analysis focuses on the behavior of producer, theory of production and finally production side. Final out come of theory of producer is supply.

PART-I

Basics of demand

1. **Concept of demand**

   i- Desire
   ii- Price
   iii- Willingness
   iv- Ability to purchase

All the commodities which has ability to satisfy humans and must have price and willingness to purchase, economic good is one which has ability to satisfy humans and have price is called "**good**" and dissatisfy is called "**bad commodity**" and not have ability to satisfy humans is called "**neutral commodity**".

"A commodity which a person would be willing and financially able to purchase at various prices and have desire is called "**Demand**".

2. **Demand function**

Factors on which demand depends

$D_X = f(P_X, P_Y, \text{Income})$

[Assume other is not measurable just like taste, habits and preferences]

3. **Determinants of demand**

   i- Own price

   Change in price of commodity leads change in demand

Δ **P_X ⬄ Δ D_X**

When ever price of commodity changes it have two effects

1. **Substitution effect**

   Commodity becomes relatively cheaper or expensive as compare to other commodities.

   |X_1| $P_{X1} = 10$ to $P_{X1} = 20$
   |X_2| $P_{X2} = 20$

2. **Income effect**

   Purchasing power of consumer change "real income" change

Case 1

Let P_X increases**:**

P_X increases ⬄ **Real income** decreases ⬄ **D_X** decreases

Income effect will be negative (-)

P_X increases ⬄ **relatively** expensive as compare to X_2 ⬄ **D_X** decreases

Substitution effect will be negative (-)

Price effect = Income effect + Substitution effect

(-) = (-) + (-)

Positive Relationship [+ shows Normal Good]

Px increases ⇔ **Real income** decreases ⇔ **D**x decreases

Negative Relationship

- Normal commodity has negative (-) relationship with price and positive (+) with real income

Negative Relationship [- shows Inferior Good]

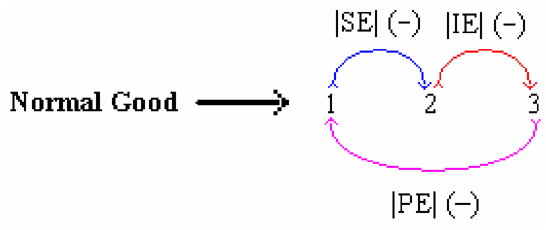
Px increases ⇔ **Real income** decreases ⇔ **D**x decreases

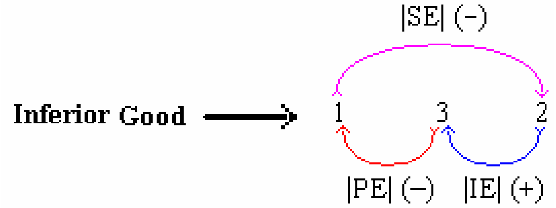
Positive Relationship

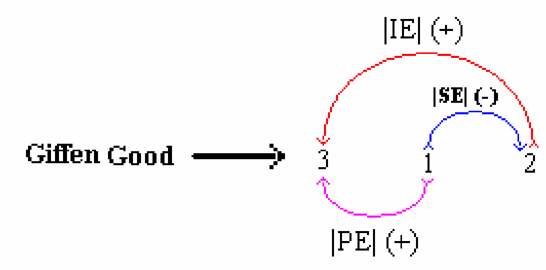
- Inferior commodity has positive (+) relationship with price and negative (-) with income

Income effect + Substitution effect = Price effect

Normal Good	**(-)**	+	**(-)**	=	**(-)**
Inferior Good	**(+)**	+	**(-)**	=	**|IE| > |SE| (+) Giffen** **|IE| <|SE| (-) Inferior**







ii- Cross price

Change in P_{X1} will leads to change in demand of X commodity or may not change

Δ **P_Y** ⇔ Δ **D_x** or $\Delta \overline{D_x}$

Case 2

Let P_X increases:

i. D_x decreases ⇔ **Complimentary** goods (use together)

Petrol price increases ⇔ **Demand for cars** decreases (complimentary goods)

ii. D_x increases ⇔ **Substitutes** goods (In place of others)

Sweater price increases ⇔ **Demand for coats** increases (substitute goods)

iii. D_x increases ⇔ **unrelated** goods (Neutral goods)

Milk price increases ⇔ **Demand for shoe** constant (unrelated goods)

iii- Income of consumer

Let Income (Y) increases:

Positive Relationship [+ shows Normal Good]

Y increases ⇔ **D_x** increases

Negative Relationship [- shows Inferior Good]

Y increases ⇔ **D_x** decreases (low)

Negative Relationship [- shows Giffen Good]

Y increases ⇔ **D_x** decreases (high)

Conclusion:

1. Own Price with demand
   - **(+)** Giffen/Strong inferior
   - **(-)** Normal good
2. Cross Price with demand
   - **(+)** Substitute goods
   - **(-)** Complimentary goods
3. Income with demand
   - **(+)** Normal goods
   - **(-)** Inferior good

Concept of elasticity of demand

To show the exact change (% change is elasticity) in demand due to any factor (Price, Income, cross price).

Price Elasticity of demand

Price elasticity of demand is percentage change in demand of a commodity due to one percentage change in its own price of commodity.

$$\varepsilon_d = \frac{\%\text{ change in demand}}{1\%\text{ change in own price}}$$

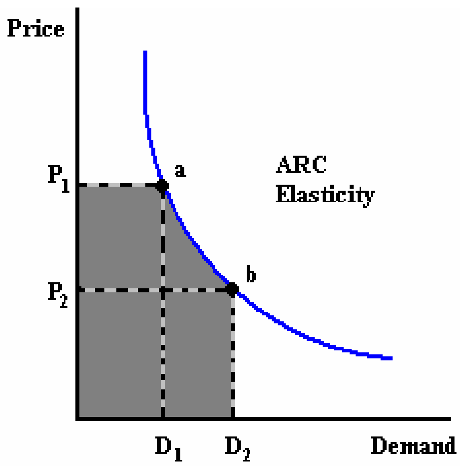
$$\varepsilon_d = \frac{\left[\frac{D_2 \quad D_1}{D_1}\right] x100}{\left[\frac{P_2 \quad P_1}{P_1}\right] x100} \qquad \Delta D = D_2 - D_1 \qquad \Delta P = P_2 - P_1$$

$$\varepsilon_d = \frac{\frac{\Delta D}{D_1}}{\frac{\Delta P}{P_1}}$$

$$\varepsilon_d = \frac{\Delta D}{\Delta P} \cdot \frac{P_1}{D_1}$$

<u>ARC Elasticity</u>

In ARC elasticity we measure the elasticity between the two points. In ARC elasticity of demand we take the average of two prices and average pf two demands between two points.

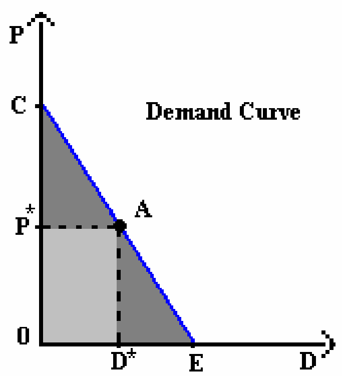


$$\text{ARC Elasticity} = \frac{\Delta D \Big/ \frac{D_1 + D_2}{2}}{\Delta P \Big/ \frac{P_1 + P_2}{2}}$$

$$= \frac{\Delta D}{\Delta P} \cdot \frac{P_1 + P_2}{D_1 + D_2}$$

Elasticity along Demand curve

A point along linear demand curve



At point "A" elasticity;

$$\varepsilon_d = \frac{\Delta D}{\Delta P} \cdot \frac{P_1}{D_1}$$

ΔD =? $\Delta D = 0D$ [ΔD = existing demand because at 0C price demand is zero]

ΔP =? $\Delta P = 0C - 0P = CP$

P = 0P

D = 0D

$$\varepsilon_d = \frac{0D}{CP} \cdot \frac{0P}{0D}$$

$$= \frac{0P}{CP}$$

$$\text{As } 0P = AD \Leftrightarrow \varepsilon_d = \frac{AD}{CP}$$

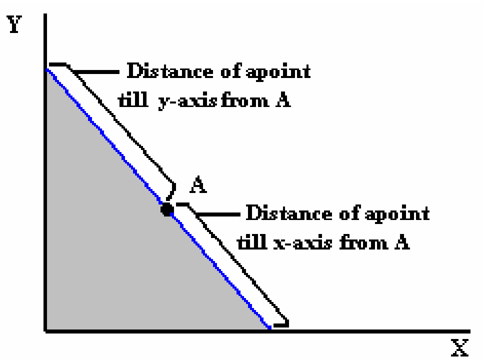
▲APC and ▲ADE are similar triangles and similar triangles have the following properties

Ratio of Base = Ratio of Perp: = Ratio of Hyp:

$$\frac{DE}{PA} = \frac{AD}{CP} = \frac{AE}{CA}$$

$$\varepsilon_d = \frac{DE}{PA} = \frac{AD}{CP} = \frac{AE}{CA}$$

$$\varepsilon_d = \frac{\text{Distance of a point till x-axis from A}}{\text{Distance of a point till y-axis from A}}$$



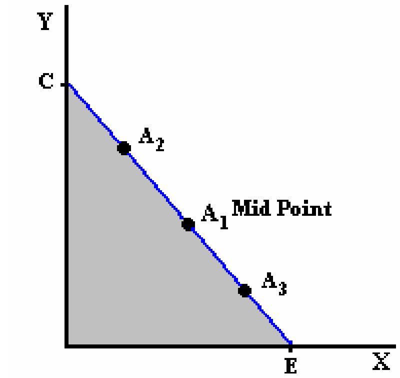
Few Implications

Take a point in mid of the curve and two other points above and below the mid point of the curve.

ε_d at $A_1 = \frac{A_1E}{A_1C}$ ⇔ $\varepsilon_d = 1$

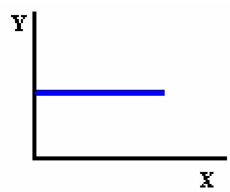
ε_d at $A_2 = \frac{A_2E}{A_2C}$ ⇔ $\varepsilon_d > 1$

ε_d at $A_3 = \frac{A_3E}{A_3C}$ ⇔ $\varepsilon_d < 1$



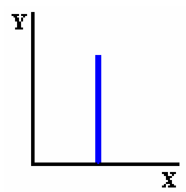
Demand curve is parallel to x-axis then elasticity is infinite

ε_d = Infinite
Slope = zero



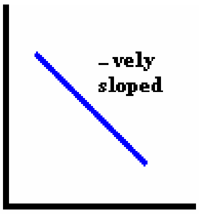
Demand curve is parallel to y-axis then elasticity is zero

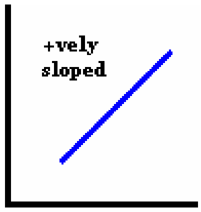
ε_d = Zero
Slope = Infinite

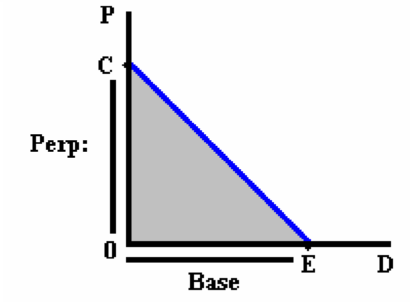


Slope of Straight Line

$$\text{Slope} = \frac{\text{Run}}{\text{Rise}} = \frac{\text{Perpendicular}}{\text{Base}}$$





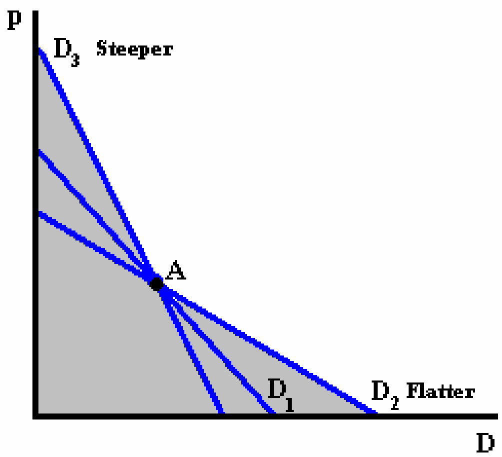


$$\text{Slope} = \frac{\text{Perp:}}{\text{Base}} = \frac{-\Delta P}{\Delta D}$$

$$\varepsilon_d = \frac{1}{\text{slope}} \cdot \frac{P}{D}$$

$$\text{Because } \frac{\Delta D}{\Delta P} = \frac{1}{\text{slope}} \qquad \text{Where slope} = \frac{\Delta P}{\Delta D}$$

$$\varepsilon_d = \frac{\text{Distance of a point till demand axis}}{\text{Distance of a point till Price axis}}$$



ε_d at point "A"

Along demand curve D_1 ⇔ $\varepsilon_d = 1$

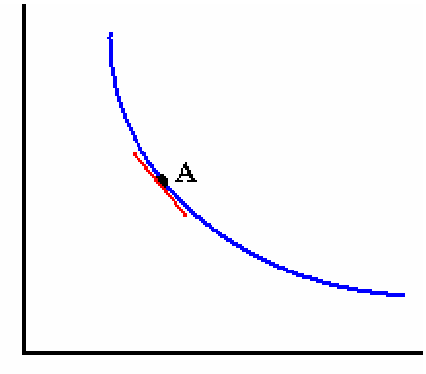
Along demand curve D_2 ⇔ $\varepsilon_d > 1$

Along demand curve D_3 ⇔ $\varepsilon_d < 1$

Conclusion:

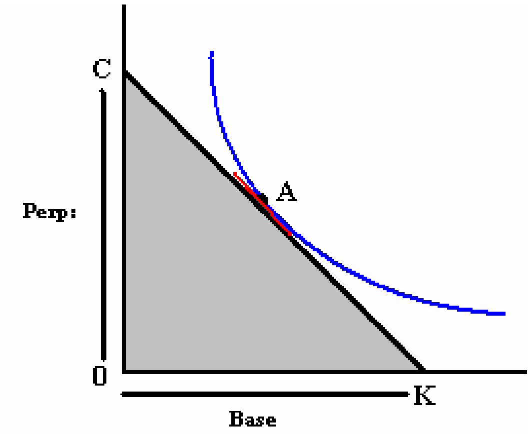
1. Flatter the demand curve greater would be the elasticity
2. Steeper the demand curve lesser would be the elasticity
3. At flatter demand curve slope will be lesser because denominator is greater than numerator
4. Greater the slope less would be the elasticity
5. Lesser the slope greater would be the elasticity

Elasticity of demand along Non linear demand curve

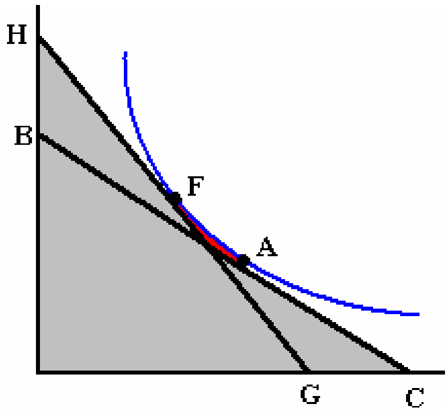


Slope at a point on a non linear curve

First we make tangent and then scale a line till both axis and then take slope by giving name to axis.



Slope at "A" = $\frac{0C}{0k}$ $\qquad$ ε_d at "A" = $\frac{AK}{AC}$



ε_d at "A" point = $\frac{AC}{AB}$

ε_d at "A" point = $\frac{FG}{FH}$

Income Elasticity of demand

Income elasticity of demand is the proportionate change in demand due to proportionate change in consumer's income.

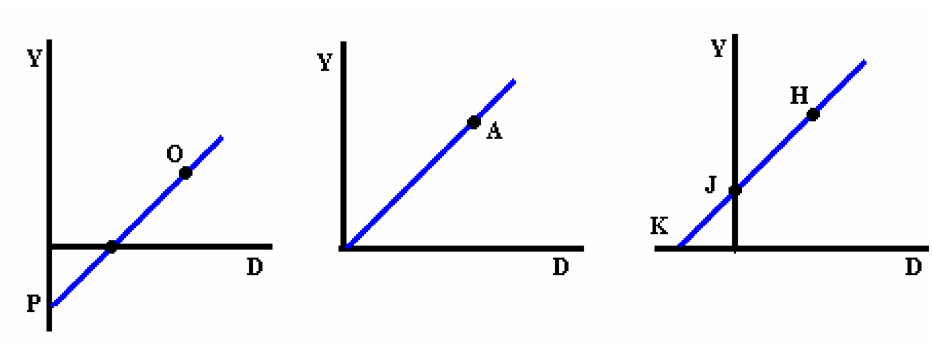
$$\varepsilon_y = \frac{\text{Proportionate change in demand}}{\text{Proportionate change in income}}$$

$$\varepsilon_y = \frac{\Delta D}{\Delta Y} \cdot \frac{Y}{D}$$

Along a Linear Engel curve

$$\varepsilon_y = \frac{\text{Distance of a point till demand axis}}{\text{Distance of a point till income axis}}$$

1. Any Engel curve starts from Y-axis ⇔ $\varepsilon_y > 1$
2. Any Engel curve starts from X-axis ⇔ $\varepsilon_y < 1$
3. Any Engel curve starts from Origin ⇔ $\varepsilon_y = 1$



$\varepsilon_y = \frac{0A}{0P}$	$\varepsilon_y = \frac{0A}{0A}$	$\varepsilon_y = \frac{HK}{HJ}$
0P > 0A,	0A = 0A	HJ < HK
=> $\varepsilon_y < 1$	=> $\varepsilon_y = 1$	=> $\varepsilon_y > 1$

Elasticity of Supply

Supply depends upon price. Price elasticity of supply is proportionate change in supply due to proportionate change in price.

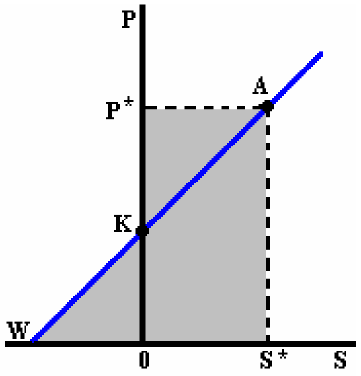
$$\varepsilon_s = \frac{\text{Proportionate change in supply}}{\text{Proportionate change in price}}$$

$$\varepsilon_s = \frac{\Delta S}{\Delta P} \cdot \frac{P}{S}$$

$$\varepsilon_s = \frac{1}{\text{Slope}} \cdot \left[\frac{P}{S}\right]$$

Along a linear supply curve

$$\varepsilon_s = \frac{\text{Distance of a point till supply axis}}{\text{Distance of a point till price axis}}$$



ε_s at point "A"

$$\varepsilon_s = \frac{\Delta S}{\Delta P} \cdot \frac{P}{S}$$

P = 0P ΔP = 0P – 0K = PK
S = 0S ΔS = 0S – 0 = 0S

$$\varepsilon_s = \frac{0S}{PK} \cdot \frac{0P}{0S}$$

$$\varepsilon_s = \frac{0P}{PK} \quad \text{as } 0P = AS$$

▲ASW and ▲APK are similar triangles and similar triangles have the following properties

Ratio of Base = Ratio of Perp: = Ratio of Hyp:

$$\frac{SW}{AP} = \frac{AS}{PK} = \frac{AW}{AK}$$

$$\varepsilon_s = \frac{\text{Distance of a point till supply axis}}{\text{Distance of a point till price axis}}$$

$$\varepsilon_s = \frac{AW}{AK}$$

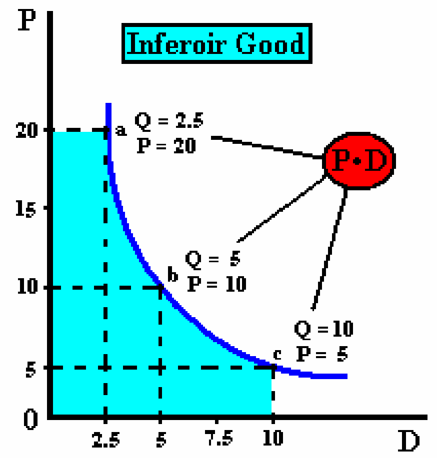
NB : - **There are three suppliers of X good, at existing price of 20 per unit, one supplier is willing to supply 100 units, second supplier is willing to supply 90 units and third supplier 70 units, however no one willing to supply at zero price, which among the three suppliers has maximum elasticity?**

Answer: Since all the supply curves are linear starting from origin therefore elasticity of each supply is equal to one ($\varepsilon_S = 1$)

A Special demand curve

A special demand curve along which price elasticity of demand is equal at all points, which ever tangent you are going to make price and quantity product remains same at all points.

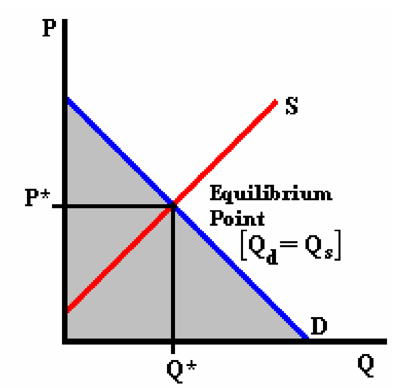
1. Product (P x Q) same at all points
2. Ratio $\frac{P}{Q}$ not same at all points



Unitary elastic curve, there is only one curve (graph) for inferior good

<u>Uses of Elasticity</u>

- **Impact of per unit tax on consumer and producer**

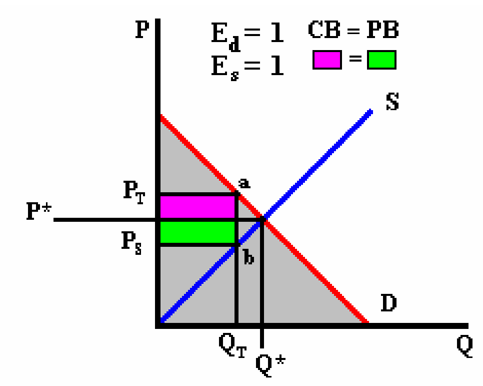


Per unit tax on commodity:

i- Entire burden bear by consumer
ii- Entire burden bear by producer
iii- Entire burden bear by both equally
iv- More burden bear by consumer
v- More burden bear by producer

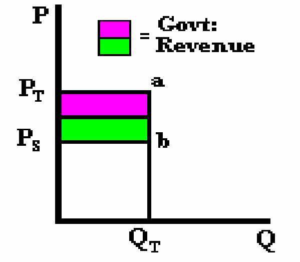
These all conditions depends upon slopes of the elasticity, determined who bear more burden of tax.

Case 1:



After Per Unit Tax

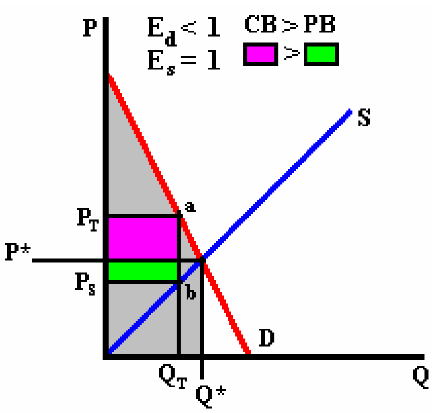
P*= before tax take by consumer
P_T = Price paid by consumer
P_S = Price taken by producer
P_T –P_S = per unit tax taken by Government
Q_T = Quantity sold after tax
Govt: Revenue = Q_T x (P_T–P_S)
= (P_T a b P_S)



In case 1, consumer and producer burdens are equal because of the same elasticity.

Case 2:

After Per Unit Tax



P*= before tax take by consumer
P_T = Price paid by consumer
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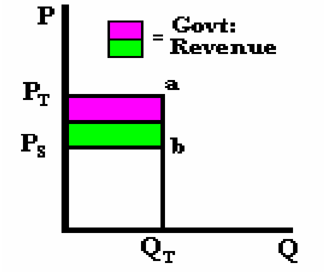
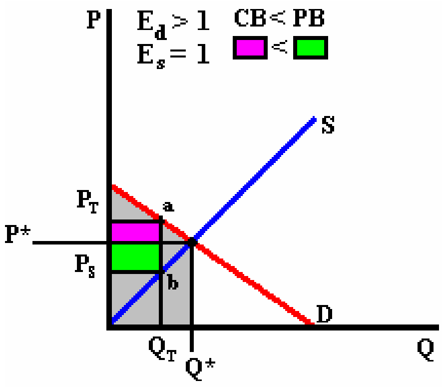


- In case 2, consumer burden is more than producer burden because of elasticity of demand is less than one.

Case 3:



After Per Unit Tax

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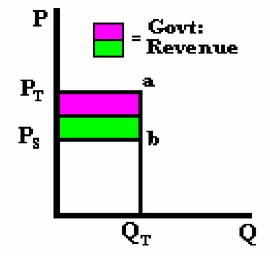
Govt: Revenue = Q_T x (P_T–P_S)
= (P_T a b P_S**)**



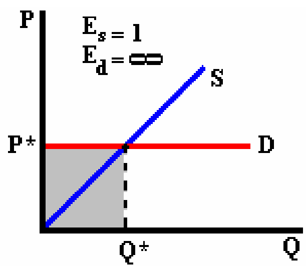
In case 3, producer burden is more than consumer burden because of elasticity of demand is greater than the elasticity of supply.

Conclusion:

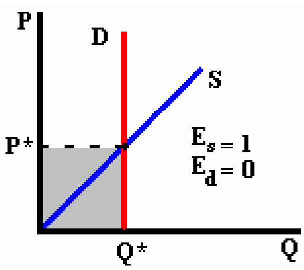
1. greater the elasticity less will be the tax burden
2. lesser the elasticity greater will be the tax burden
3. if elasticity is same for demand and supply both bear the equal burden

Assignment:

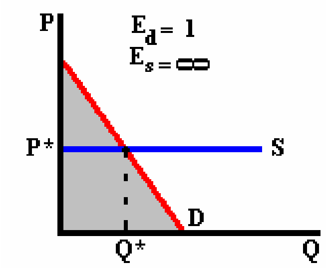
1. Demand curve is parallel to quantity axis and supply curve is positively sloped.



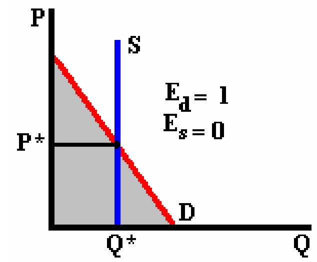
2. Demand curve is parallel to price axis and supply curve is positively sloped.



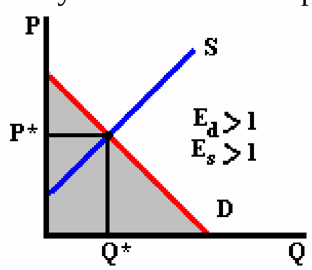
3. Supply curve is parallel to quantity axis and demand curve is negatively sloped.



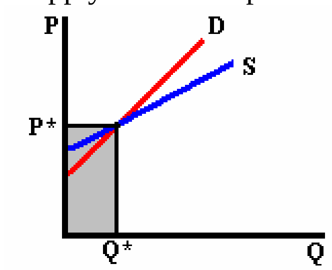
4. Supply curve is parallel to price axis and demand curve is negatively sloped.



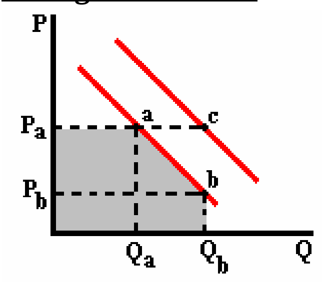
5. At equilibrium elasticity of demand and supply is greater than one.



6. Both demand and supply curves are positively sloped.



Change in Demand

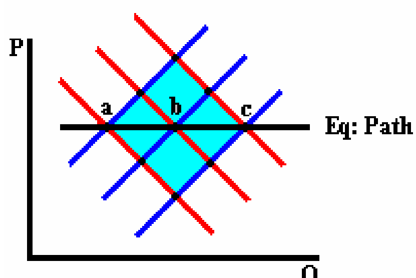


- Change in demand due to change in own price there will be movement along the demand curve.
- Change in demand due to factors other than price the demand curve would shift.

Two type changes:

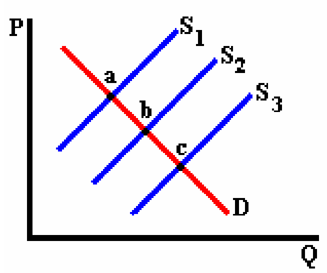
- Along the demand curve due to own price
- Other effect can shift demand curve due to factors other than price

Identification of Demand and Supply



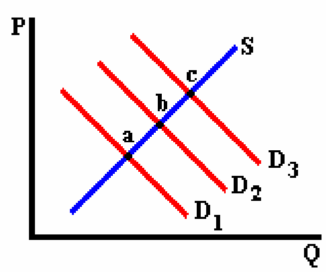
None

- If your equilibrium path is on demand curve is called demand identified.



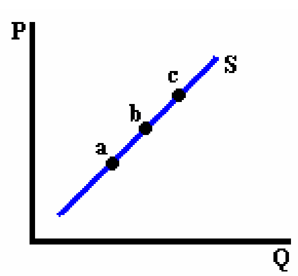
Demand is identified

- If your equilibrium path is on supply curve is called supply identified.

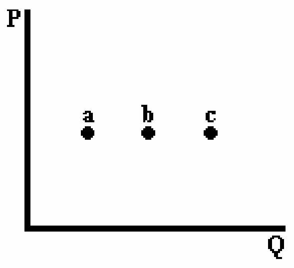


Supply is identified

Identified is one along all the equilibrium points are present.



Demand positively sloped	Giffen commodity
Demand negatively slope	Normal commodity



We can say about anything

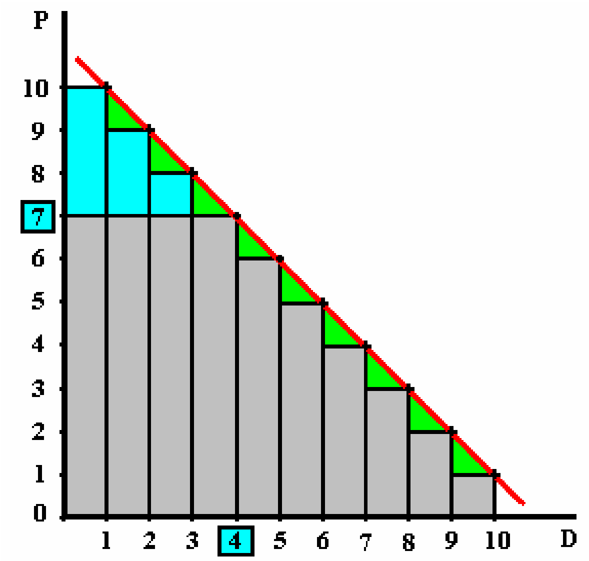
Concept of Consumer and Producer Surplus

Consumer surplus:

Consumer surplus is the area below demand curve and above price line, a portion below the demand curve above price line. Difference between what consumer willing to pay and what he actually pays

Along a demand curve

Price	*Demand*
10	1
9	2
8	3
"7"	"4"
6	5
5	6



When taken unique Price and demand expenditure will be

7 x 4 = 28

Seller directly offer price of 7 for 4 quantities. What ration seller do, he should charge 10 for 1 commodity.

Seller tries to get more profit as compare to consumer by taking (individual charges) pricing by parts.

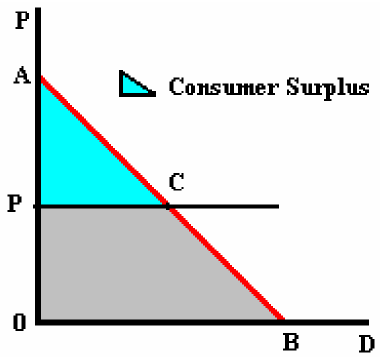
Price	Q. Purchased	Expenditure
10	1	10 x 1 = 10
9	1	9 x 1 = 9
8	1	8 x 1 = 8
7	1	7 x 1 = 7
Total	**Q_d = 4**	**T.E = 37**

Unique Price 7 x 4 = 28
Pricing by parts 34

Consumer surplus = P.P – U.P
= 34 – 28
= 8

Conclusion:

1. Taking unique price is good for consumer
2. Taking pricing by parts is not good for consumer's point of view



▲ APC shows the area above price line and below the demand curve.

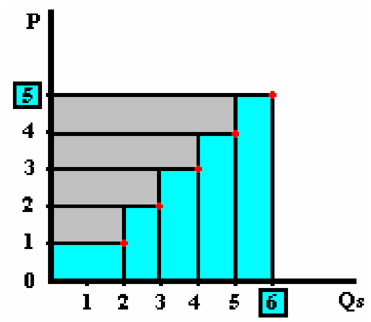
Producer Surplus

"**Producer** surplus is the area above the supply curve and below the price line" or "a portion above the supply curve and below the price line". Price (amount) which producer gets above the minimum acceptable (reserved) price.

Let a person wants to sale a car at 400000 and his car sold at 420000, extra 20000 is his surplus.

Along a Supply curve

Price	Supply
1	2
2	3
3	4
4	5
"5"	"6"



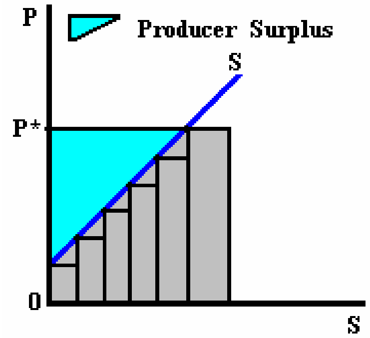
Unique price

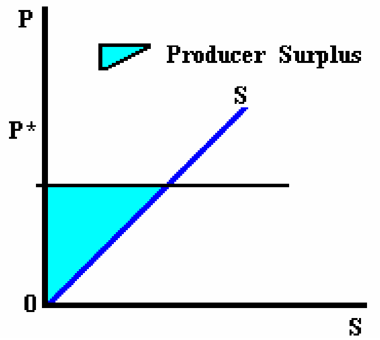
P = 5 Revenue = P x Q_s
Q_s = 6 = 5 x 6
= 30

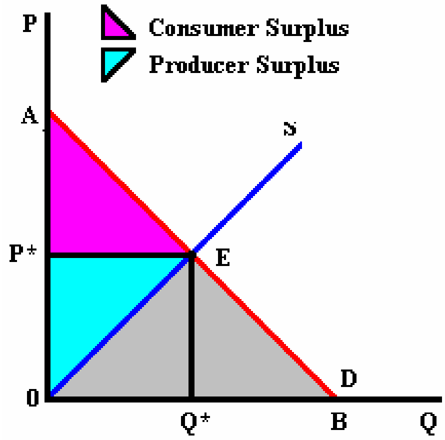
Price	*Quantity supply*	*Revenue*
1	2	1 x 2 = 2
2	1	2 x 1 = 2
3	1	3 x 1 = 3
4	1	4 x 1 = 4
5	1	5 x 1 = 5
Total	**Q_s = 6**	**Total Revenue = 16**

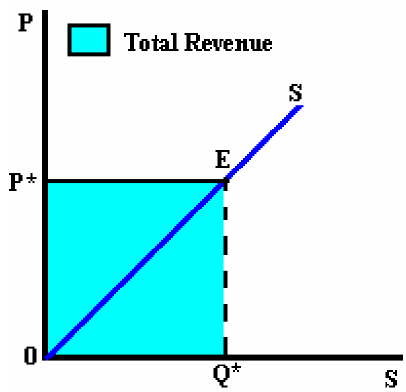
Pricing by parts total revenue = 16
Unique price total revenue = 30

Producer Surplus = **30 – 16**
= **14**

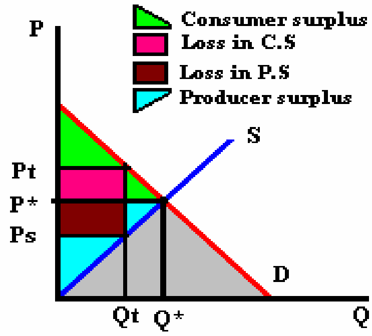




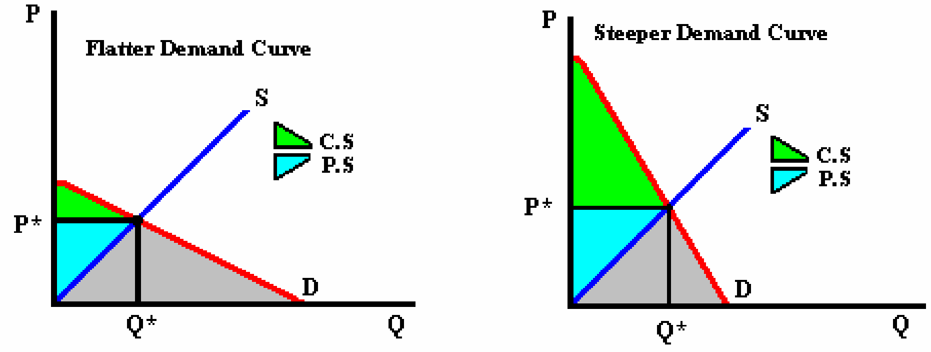




Per Unit Tax and Consumer Surplus and Producer Surplus



How consumer surplus and producer surplus are related to demand and supply curves

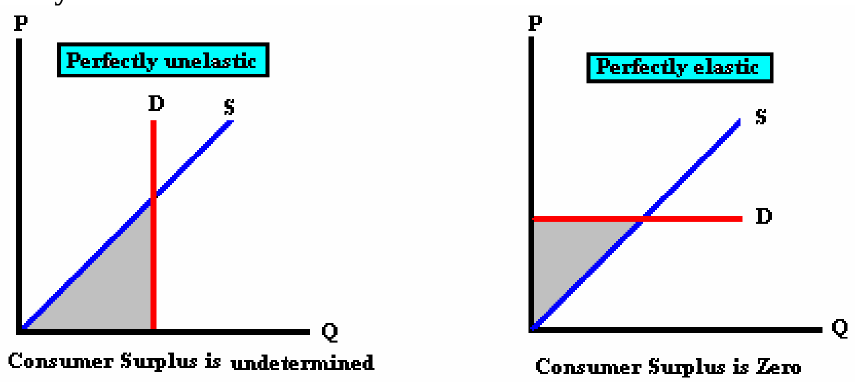


1. Flatter the demand curve consumer surplus will be less
2. Steeper the demand curve consumer surplus will be greater

<u>Assignment:</u>

Q.1: What would be consumer surplus in the following cases?

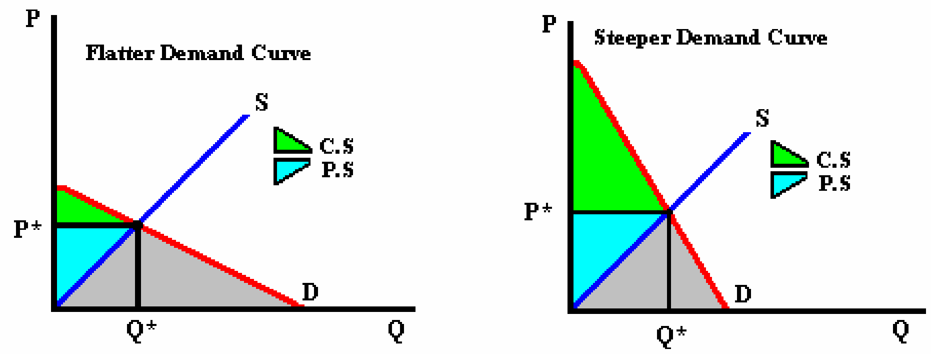
- **i. Perfectly elastic demand curve**
- **ii. Perfectly un-elastic demand curve**



Q.2: Consider a 45° positively slope linear supply curve, show that at equilibrium;

- **i.** Price elasticity of supply is greater than elasticity of demand.
- **ii.** Elasticity of supply is less than elasticity of demand.

Discuss the relative proportion of consumer and producer surplus in each case?



PART-II

Theory of Consumer Behavior

Consumer is one who spends some of his money (income) for consumer goods, things for consumption or for personal use and satisfaction purpose i.e. Car for personal use.

Issue: How a rational consumer would make a consumption decision?
We know always one problem that limited resources (limited income).

Desire ⇔ Need ⇔ Demand

Income always limited, their resources are limited, and how a consumer will make the decision to spend on consumer goods, best use of limited income on different consumer goods to get maximum satisfaction.

1. All consumer has limited income
2. Object is to get maximum satisfaction
   - i. Consumer preferences
   - ii. Budget constraints
   - iii. Consumer choices

What consumer wants to purchase, which good he like and dislike and which good is in his consumption plane (most urgent preference). We can not fulfill all its desires. In consumer preferences ranking is essential part and limitations are imposed in the shape of budget constraints due to limited income.

1. A list of commodities which consumer wants to purchase, endless list (huge list of commodities).
2. Looking at purchasing power.
3. Make choices among selected goods due to budget constraints.

1- Consumer Preferences

Concept of Utility

Utility is actually ability of any thing to satisfy human beings, root cause of all consumption or economic activities. How we can measure or express utility? There are two approaches:

- I. Cardinal Approach (utility in quantitative units)
- II. Ordinal Approach (utility can be ranked)

Cardinal Approach

This approach expresses utility in measurable numbers or in quantitative units (ways). There is a unit of measurement called **Util**. There are two views of cardinal approach;

i- Utility is Additive

Utility of all comparable goods can be aggregated, if an orange gives 5 Utils of utility and an apple gives 6 Utils of utility then the both an orange and apple gives 5 plus 6 equal to 11 Utils of utility.

ii- Utility is Non-additive

Utility is measurable but not additive that it depends simultaneously on all amounts of the different goods consumed.

Marginal Utility (MU)

Marginal utility is the change in total utility due to additional unit of consumption.

$$MU = \frac{\Delta \text{ Total Utility}}{\Delta \text{ Q (One unit)}}$$

- If more than one unit consumed then we can not measure marginal utility and only on every additional unit

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y} \quad \textbf{Equilibrium condition}$$

$$\boxed{\frac{MU_x}{MU_y} = \frac{P_x}{P_y}}$$

Where as $\frac{MU_x}{MU_y}$ it is personal evaluation, it is not same for all consumers.

Where as $\frac{P_x}{P_y}$ it is Market evaluation, it is same for all consumers. Prices are determined in market.

It is based on imaginary numbers (filiation's numbers), it may be end up with wrong findings it is totally rejected approach.

Ordinal Approach

Utility can not be expressed in quantitative measures or units like Utils, however its ranking is possible when comparison is possible. Consumer goods are arranged and depending upon preferences.

Axioms of Consumer Preferences

i- Completeness

Consumer can rank utility of different goods according to their consumption bundle, ability to prefer;

$$A \sqsupseteq B \quad \text{or} \quad B \sqsupseteq A \quad \text{or} \quad A \approx B$$

If we are given different consumption bundles, individual has ability to ranking. If "A" preferred (ב) over "B" then all consumption bundles closer to "A" would also preferred over "B", they are clearly different.

ii- Continuity

It means that consumer has to prefer one good over other will remains constant till period of analysis.

iii- Consistency

It means that consumer should be consistent it means that indifference curve never intersect each other.

iv- More is preferred over less

It means that if "A" preferred over "B" then all consumption bundles closer to "A" would also preferred over "B", they are clearly different.

Indifference Curve (IC)

"A curve showing various combinations of two commodities which are equally preferred by the consumer, their satisfaction or utility level is same along that curve". Consumer has ability to rank, ranking is possible if we show all consumption bundles on the graph, and all points are equally preferred. Utility along indifference curve is same therefore it is also called **"ISO utility"** curve.

For the sack of analysis there are some limitations at all time, only two axis are taken on graph. Assume consumer only consuming two commodities only on graph, one measure along x-axis and other on y-axis.

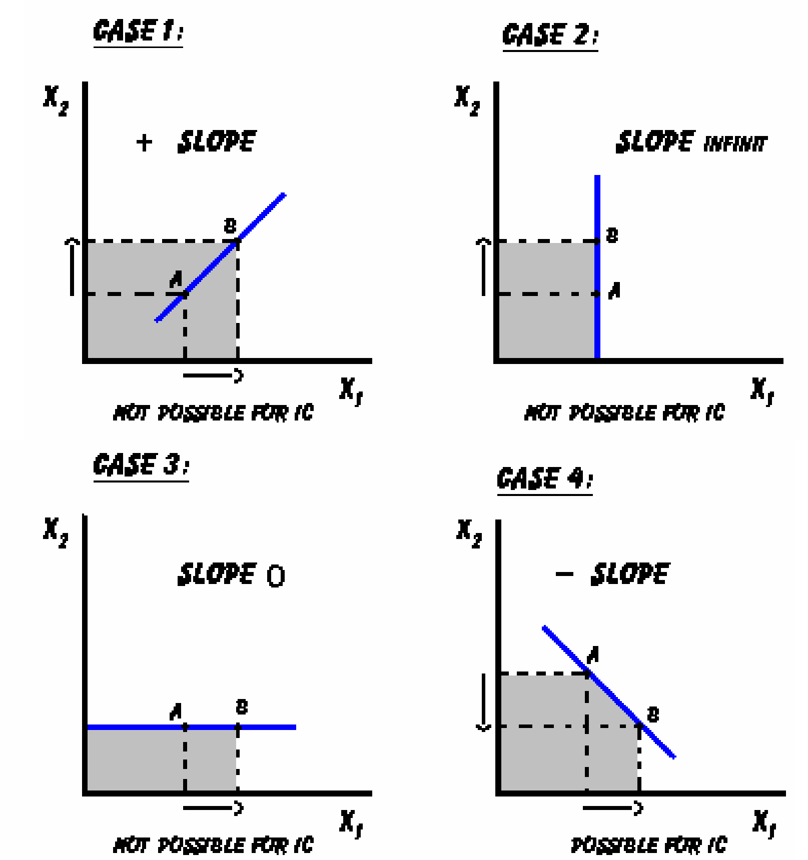
Along An Indifference Curve

$$\mathbf{U = f(X_1, X_2) \Leftrightarrow k}$$

N.B: - We are going to increase both commodities then utility will remains same, it is not possible to prefer neither increase nor decrease both commodities at the same time.

a) Along IC it is not possible to increase both commodities together or vise versa
b) Along IC it is not possible to change the quantity of one commodity by keeping other constant.
c) Along IC if one commodity increased then for same utility it is necessary to decrease the other commodity.

Gain in utility from increasing one = Loss in utility from decreasing other



In fact above three cases for IC, they are not possible because they can not fulfill the fourth axiom of consumer preference (More is preferred over less) but in fourth case it is possible because if we increase one at same time we are also decreasing the other commodity with same proportion.

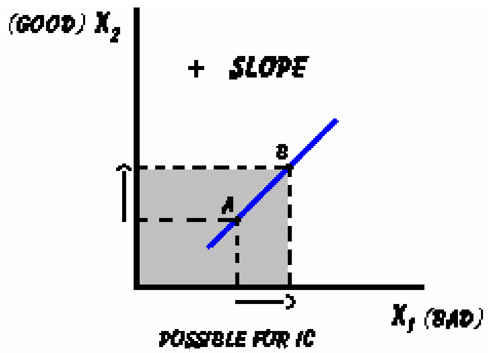
<u>From "B" to "A"</u>

X_1 decreasing   Commodities are perfect substitutes (give and take principle)

X_2 decreasing

<u>Few Special Cases:</u>

<u>Case 1:</u>



X_1 increasing (Bad Commodity) utility loss.

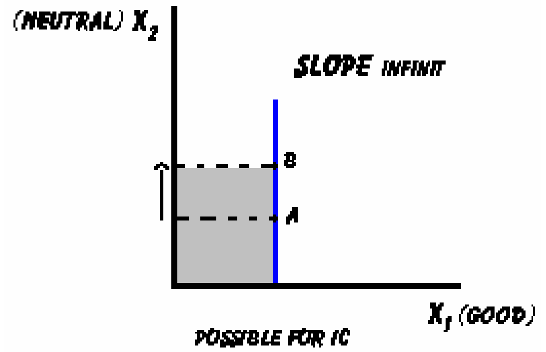
X_2 increasing (Good Commodity) utility benefit (compensated).

<u>Starting point "A"</u>

Loss due to increase X_1 = Gain due to increase X_2

When one commodity is Good commodity and other is Bad commodity then slope will be positive.

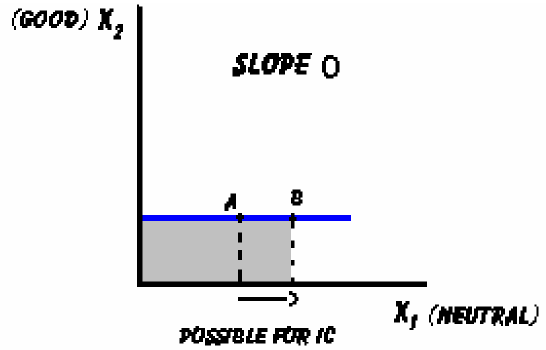
Case 2:



X_2 increasing consumption utility should increase

X_1 constant

Case 3:



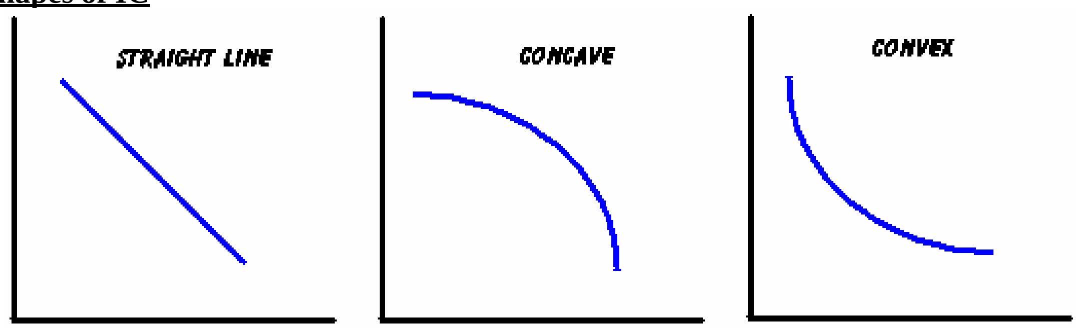
X_1 increasing consumption utility should increase

X_2 constant

When one commodity is neutral, IC will be parallel to x-axis or y-axis.

- Axiom number four satisfy negatively sloped IC
- Axiom number four do not satisfy positively, vertically and horizontal sloped IC's

Shapes of IC



Shape of IC generally convex but above two are also valid. Indifference curves may be concave, convex or straight line, if IC is negatively sloped.

Marginal Rate of Substitution (MRS)

It is the number (units) of one commodity which consumer is willing to give up and having an additional one unit of other commodity. Such that consumer remains on the same IC. MRS is always negative.

We assume

- Two commodities
- Perfect substitutes
- Satisfaction level is constant (IC is given)

Let X_1 and X_2 be the given two commodities such that consumer wants to increase X_1 and decreases X_2 then MRS of X_1 and X_2 is "the amount of X_2 which the consumer has to decrease in order to increase one unit of X_1.
Such that satisfaction level is same

$$\underset{(X_1 \text{ increases for } X_2 \text{ decreases})}{\text{MRS of } X_1 \text{ for } X_2} = \frac{\Delta X_2}{\Delta X_1}$$

<u>**MRS of X_1 for X_2 has three different possibilities**</u>

1) MRS of X_1 for X_2 (Decreasing)
2) MRS of X_1 for X_2 (Increasing)
3) MRS of X_1 for X_2 (Constant)

We are discussing three possibilities now

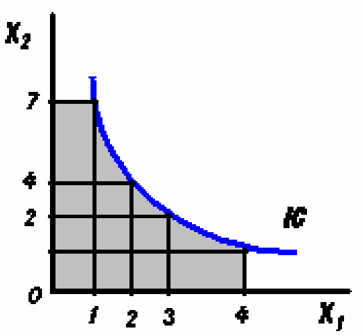
1- MRS of X_1 for X_2 (Decreasing)

X_1	X_2	MRS
1	7	---
2	4	3
3	2	2
4	1	1

MRS of X_1 for X_2 (diminishes)

<u>For first possibility</u>

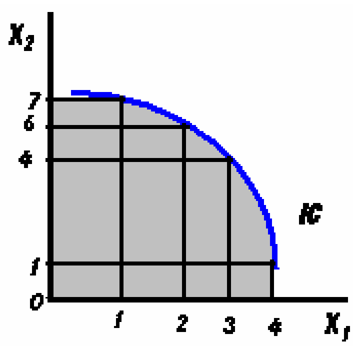
MRS X_1 increases for X_2 decreases
IC is convex to origin



2- MRS of X_1 for X_2 (Increasing)

X_1	X_2	MRS
1	7	---
2	6	1
3	4	2
4	1	3

MRS of X_1 for X_2 (increases)

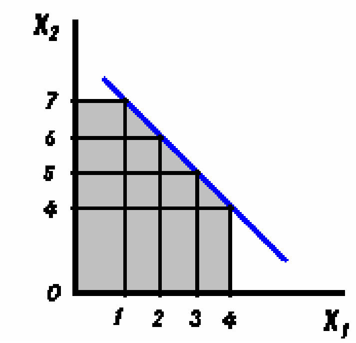


For first possibility

MRS X_1 increases for X_2 decreases
IC is concave to origin

3- MRS of X_1 for X_2 (Constant)

X_1	X_2	MRS
1	7	---
2	6	1
3	5	1
4	4	1



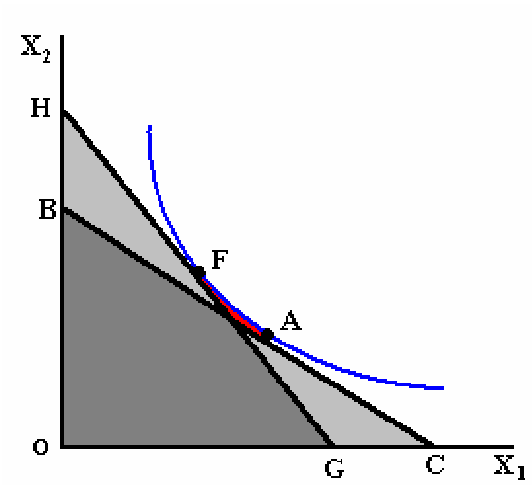
MRS of X_1 for X_2 (increases)

For first possibility

MRS X_1 increases for X_2 decreases
IC is concave to origin

Whether is it possible

Case 1:



Here MRS of X_1 for X_2 should be at point "F" greater then point "A"

$$\text{MRS of } X_1 \text{ for } X_2 \text{ at } F = \frac{\text{Perp:}}{\text{Base}} = \frac{OH}{OG}$$

$$\text{MRS of } X_1 \text{ for } X_2 \text{ at } A = \frac{\text{Perp:}}{\text{Base}} = \frac{OB}{OC}$$

Comparison

$OH > OB$

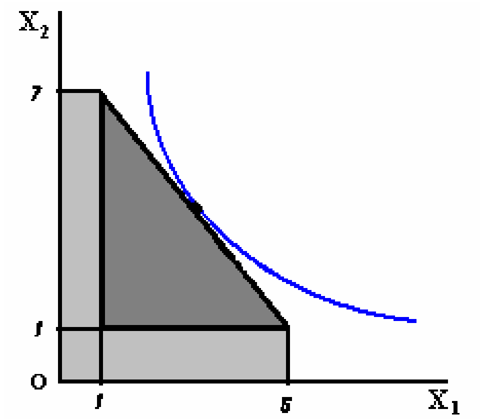
$OG < OC$

Numerator OH is greater
Denominator OC is greater

- Greater the numerator has greater value
- Greater the denominator has lesser value

MRS at point "F" greater then point "A"

Now by Mathematically



- Slope of IC is same always in graphically and mathematically

MRS = IC (graphically) = IC (mathematically)

<u>Case 2:</u>

Here MRS of X_1 for X_2 should be at point "F" greater then point "A"

$$\text{MRS of } X_1 \text{ for } X_2 \text{ at } F = \frac{\text{Perp:}}{\text{Base}} = \frac{OH}{OG}$$

$$\text{MRS of } X_1 \text{ for } X_2 \text{ at } A = \frac{\text{Perp:}}{\text{Base}} = \frac{OB}{OC}$$

<u>Comparison</u>

$OH > OB$

$OG < OC$

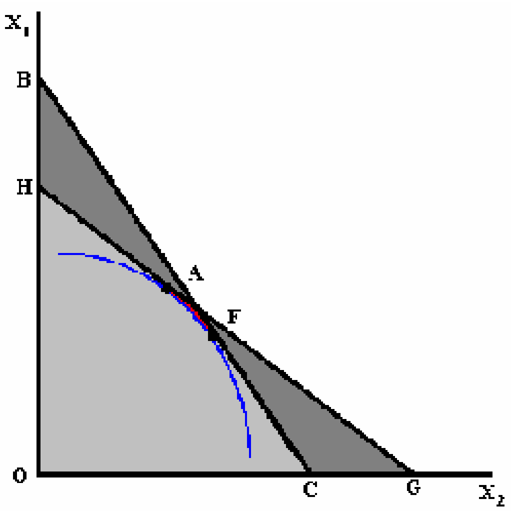
Numerator OH is greater
Denominator OC is greater

- Greater the numerator has greater value
- Greater the denominator has lesser value

MRS at point "F" greater then point "A"

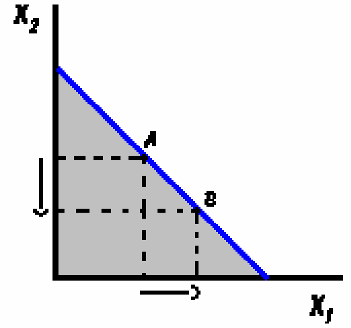
- Slope of IC is same always in graphically and mathematically

MRS = IC (graphically) = IC (mathematically)



Case 3:

MRS is constant as IC is negatively sloped so we ignore it in MRS



- MRS can not be horizontal, vertical or positively sloped because of the fourth axiom.
- MRS is possible only in negatively sloped IC
- MRS is the slope of IC
- MRS determine the shape of IC

MRS of "X" for "Y" is diminishing

- IC is convex

$\bar{U} = f(X_1, X_2)$

$$\Rightarrow \partial U = \frac{\partial U}{\partial X_1} \cdot \partial X_1 + \frac{\partial U}{\partial X_2} \cdot \partial X_2$$

∂ = Total change
∂ = Partial change

Along an IC total change is zero

$$\Rightarrow 0 = \frac{\partial U}{\partial X_1} \cdot \partial X_1 + \frac{\partial U}{\partial X_2} \cdot \partial X_2$$

$$\Rightarrow -\frac{\partial U}{\partial X_1} \cdot \partial X_1 = \frac{\partial U}{\partial X_2} \cdot \partial X_2$$

$\frac{\partial U}{\partial X_1}$ Marginal Utility of X_1

$\frac{\partial U}{\partial X_2}$ Marginal Utility of X_2

$$\Rightarrow -(MUx_1) \cdot \partial X_1 = (MUx_2) \cdot \partial X_2$$

$$\Rightarrow \frac{\partial X_1}{\partial X_2} = -\frac{(MUx_2)}{(MUx_1)}$$

$$\text{MRS of } X_2 \text{ for } X_1 = -\frac{(MUx_2)}{(MUx_1)}$$

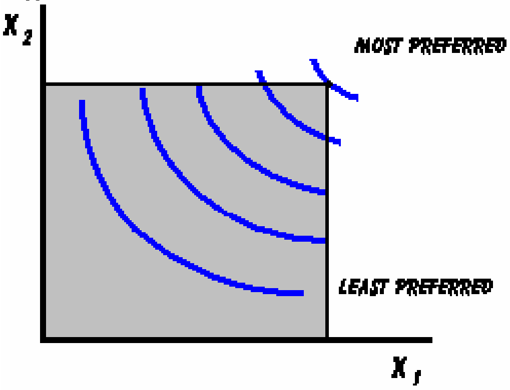
$$\text{MRS of } X_1 \text{ for } X_2 = -\frac{(MUx_1)}{(MUx_2)}$$

- MRS is also ratio of marginal utilities of the commodities

Theoretically: - Amount of X_2 which we are sacrificing to get one more additional amount of X_1.
Graphically: - It is slope of Indifference curve.
Mathematically: - It is the ratio of marginal utilities of the commodities.

2- Budget Constraints

Concept of budget constraints or line



In this graph showing various combinations of indifference curves, each high consumption bundle of IC is more preferred over less, every consumer wants to select the highest IC to get maximum satisfaction but the entire consumer have one problem that is limited income. In limited income consumer have to select that consumption bundle which will be in his reach and which gives maximum satisfaction.

"Budget constraints show how much of different commodities consumer can purchase with his limited income and at given prices"? Budget line or Price line is a tool.

Let commodities are X_1 and X_2 and income of consumer is Y (Y can be any number), then budget line would be

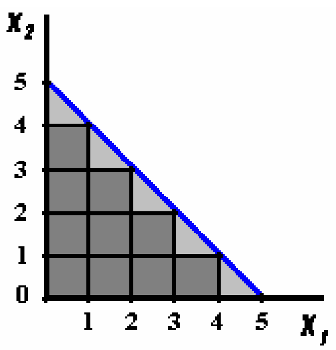
$$\mathbf{Y = (Px_1)(X_1) + (Px_2)(X_2)}$$

Y = total expenditure on X_1 and X_2
$(Px_1)(X_1)$ = expenditure on X_1
$(Px_2)(X_2)$ = expenditure on X_2

Assume that Y = 100, Px_1 = 20 per unit and Px_2 = 20 per unit

Then budget line will be

$$100 = 20X_1 + 20X_2$$
$$= 20\ (0)\ + 20\ (5)$$
$$= 20\ (1)\ + 20\ (4)$$
$$= 20\ (2)\ + 20\ (3)$$
$$= 20\ (3)\ + 20\ (2)$$
$$= 20\ (4)\ + 20\ (1)$$
$$= 20\ (5)\ + 20\ (0)$$



$$Y = (P_{x_1})(X_1) + (P_{x_2})(X_2)$$

At Intercepts

(I) $X_1 = 0$

$Y = (P_{x_1})(0) + (P_{x_2})(X_2)$

$Y = (P_{x_2})(X_2)$

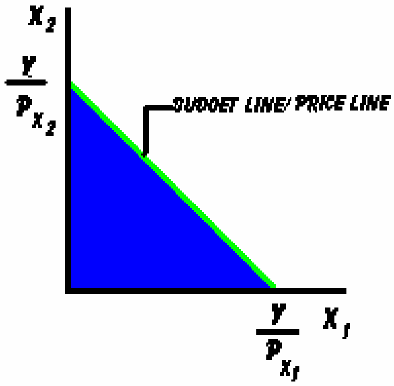
$X_2 = \frac{Y}{P_{x_2}}$

(II) $X_2 = 0$

$Y = (P_{x_1})(X_1) + (P_{x_2})(0)$

$Y = (P_{x_1})(X_1)$

$X_1 = \frac{Y}{P_{x_1}}$



Slope of Budget Line

$$\text{Slope} = -\frac{\text{Perpendicular}}{\text{Base}}$$

$$= -\frac{Y/P_{x_2}}{Y/P_{x_1}}$$

$$= -\frac{Y}{P_{x_2}} \cdot \frac{P_{x_1}}{Y}$$

$$\text{Slope} = -\frac{P_{x_1}}{P_{x_2}}$$

When prices are same slope will be same

Let

$P_{X_1} = 1$, $P_{X_2} = 5$, Slope $= -\frac{1}{5}$

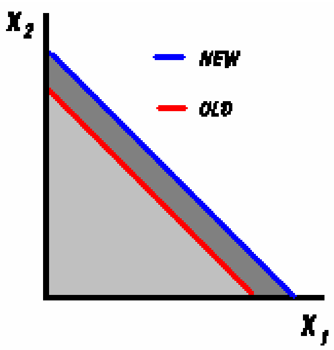
Budget Line for n-commodities

$$Y = \sum P_{X_i} X_i \qquad (i = 1, 2, 3 \ldots\ldots\ldots\ldots, n)$$

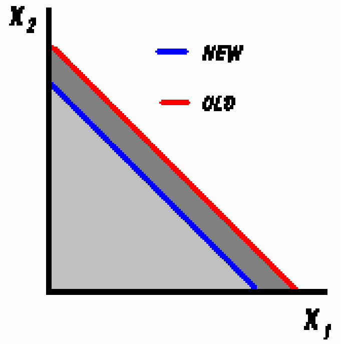
Example:

What will be the effect on budget line in the following cases?

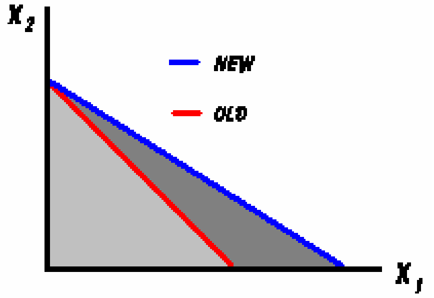
- If income of consumer increases and prices of X_1 and X_2 remains constant



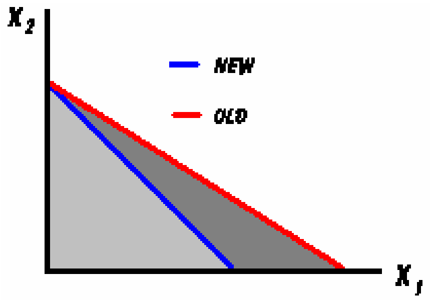
- If income of consumer decreases and prices of X_1 and X_2 remains constant



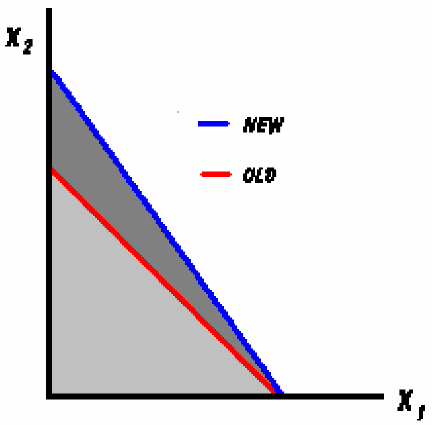
- If price of X_1 decreases and price of X_2 and income remains unchanged



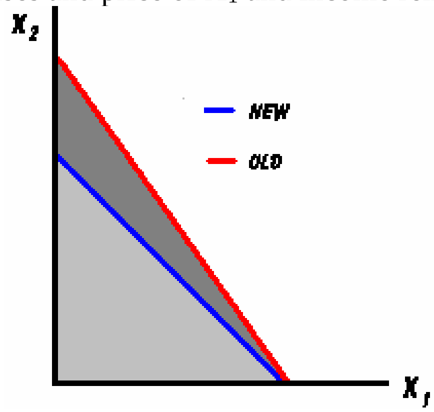
- If price of X_1 increases and price of X_2 and income remains unchanged



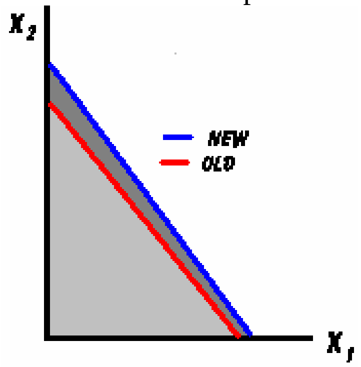
- If price of X_2 decreases and price of X_1 and income remains unchanged



- If price of X_2 increases and price of X_1 and income remains unchanged



- If price of X_1 and income increases and price of X_2 remains unchanged



- Budget line along linear straight line during analysis does not change

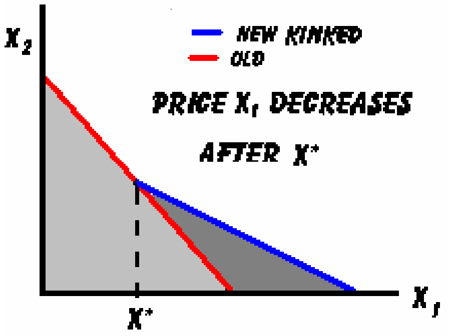
Non-linear Budget Line

Can be of two types

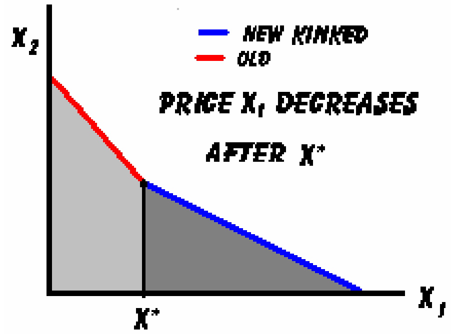
1. Kinked budget line
2. Curved budget line

Example:

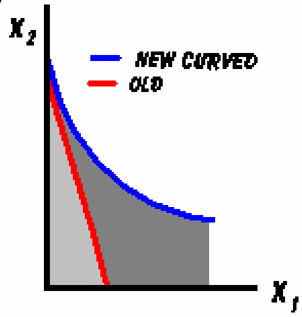
There are two commodities which are included in purchasing plane of consumer, such that prices of X_1 commodity remains same till specific limit say X_1* and after that its prices decrease by a constant amount, where as price of X_2 remains unchanged.



When prices of any commodity changes after specific amount limit or prices of commodity changes by a constant amount after specific purchase limit it make the kink in the budget line then it will be called as "kinked budget line".



If continuous change in any commodity's prices after every unit it will make curve then it will be called "curved budget line".

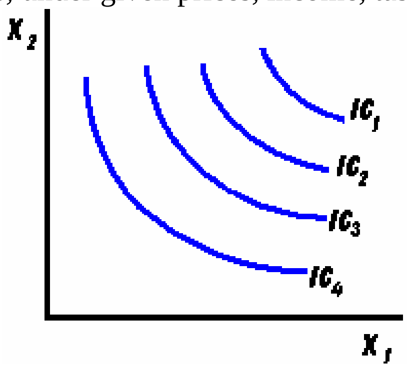


1- If every next unit of X_1 commodity get cheaper
2- If every next unit of X_1 commodity get expensive

- If we have convex budget line it means prices increases continuously
- If we have concave budget line it means prices decreases continuously

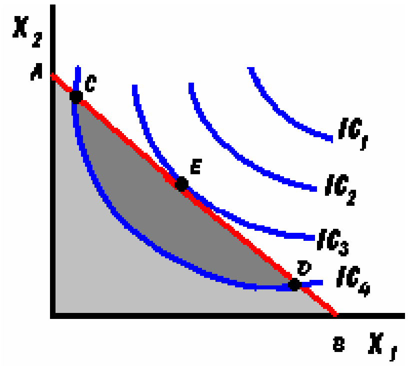
Consumer Equilibrium

Consumer equilibrium through ordinal IC approach, where satisfaction of the consumer is maximizes satisfaction dependent. A consumer is said to be in the state of equilibrium, if he gets maximum satisfaction from the purchased consumption bundle (under the given conditions, under given prices, income, taste/ preferences).



1- When IC is convex to origin/ MRS diminishes
2- When IC is concave to origin/ MRS increases
3- When IC is straight line/ MRS constant

- **When IC is convex to origin.**
  Let say we have IC map

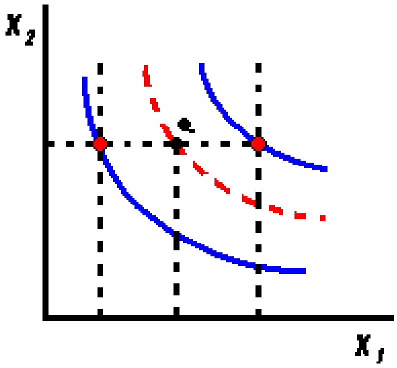


i) Indifference curves are showing preferences
ii) "AB" is the budget line showing budget constraints of consumer

An important Assumption

Commodities are divisible, scaling up the commodities because if let say commodities are divisible, what its implication of this IC.
IC is continuous and divisible

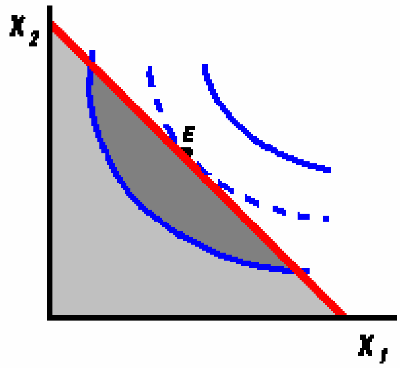
X_1	X_2
1	10
1.1	8.5
1.3	7.5
:	:
:	:
:	:
2	5



- Does there is IC in between indifference curves, not mapped and there are large numbers of IC's of different commodities in between two indifference curves.

Implication: - IC is present every where on the plane, IC always thin not thick, assume E* is the most preferred, it is equilibrium point of consumer.
Where;

Consumer level = $OX_1' + OX_2'$



- IC is present in between two indifference curves.

Condition of Equilibrium:

- Budget line must be tangent to IC.
- Slope of budget line = slope of IC

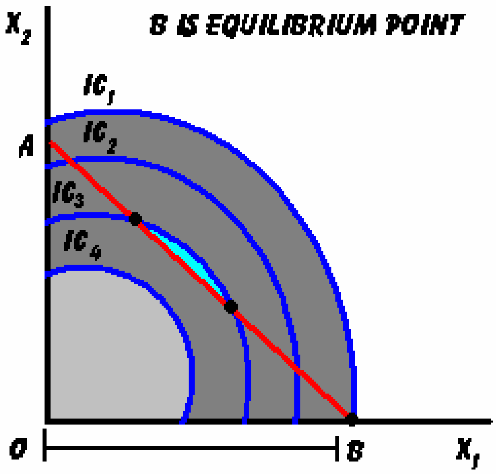
Slope of IC

$$-\frac{P_{x1}}{P_{x2}} = MRS_{x1,x2} \qquad \text{“Diminishes”}$$

$$\text{As} \quad MRS_{x1,x2} = -\frac{P_{x1}}{P_{x2}}$$

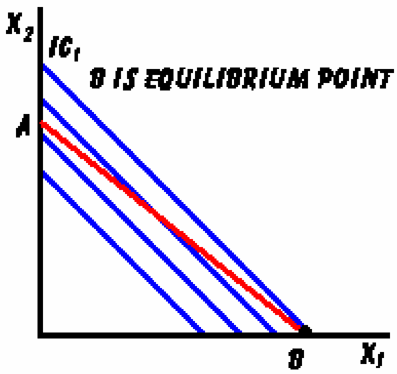
$$\frac{MU_{x1}}{MU_{x2}} = \frac{-P_{x1}}{P_{x2}}$$

- Consumer Equilibrium formula
- **When IC is concave to origin.**



At point “B” consumption level = OB and consumer will purchase only X_1 commodity. We have a corner solution; consumer can prefer to purchase/consume either X_1 or X_2 commodities, he has to consume only one commodity.

- Consumer Equilibrium point is corner solution.
- **When IC is straight line**.



IC_1 is more preferred (perfect substitutes).
Interior and exterior solution exist in consumer equilibrium point in IC

- IC one is beyond the reach of consumer and IC two is under his reach at budget line may exist also.

Shapes of IC	*Behavior of MRS*	*Equilibrium Point*
Convex	Diminishes	Interior solution
Concave	Increasing	Exterior solution
Straight Line	Constant	Exterior solution

Interior Solution: Consumer will consume both commodities and budget line must be tangent to IC.

At equilibrium

Slope of budget line = Slope of IC

Exterior Solution: Consumer will consume only one commodity and their will be corner solution.

Example: X_1 ----- 250ml bottle of Pepsi (1/4)
X_2 ----- 100ml bottle of Pepsi (1/4)

It is only in identical commodities

- MRS constant then MU also constant
- Perfectly substitute commodities then we will have always corner solution

N.B: - MRS for X_2 diminishes (IC-convex) for one additional unit of X_1, consumer would give up lesser and lesser amount of X_2.

Example:

X_1	X_2	MRS
1	20	-----
2	15	5
3	11	4
4	8	3

- MU always diminishes general law exception satisfies is last unit of commodity consumed
- MU both commodities diminishes, MRS also diminishes, MU for X_1 decreases and MU for X_2 increases only consume

i. If marginal utilities of both commodities are diminishes then IC would be convex to origin

ii. If marginal utility of one commodity increases and other is decreases then IC is concave

iii. If both commodities have same marginal utility then IC is straight line

Question: If there are two commodities and X_1 decreases then marginal utility also decreases and X_2

Increases then it marginal utility also increases which would be the IC convex, concave or straight line and corner solution, X_2 consumed more? (Money, Education's marginal utility increases)

Most Realistic case

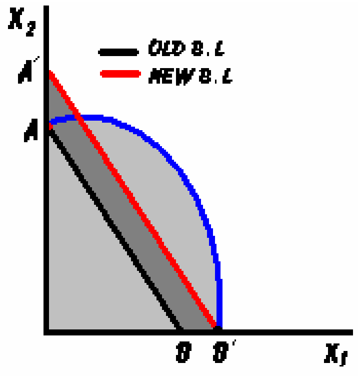
- Marginal utility increases in rare cases
- Marginal utility decreases in identical commodity cases

Bench Mark (Conclusion)

- Indifference curves of normal commodities are always convex to origin, interior solution is made and both have diminishing marginal utilities.

Conditions:

- Slope of budget line = slope of IC
- IC convex to origin



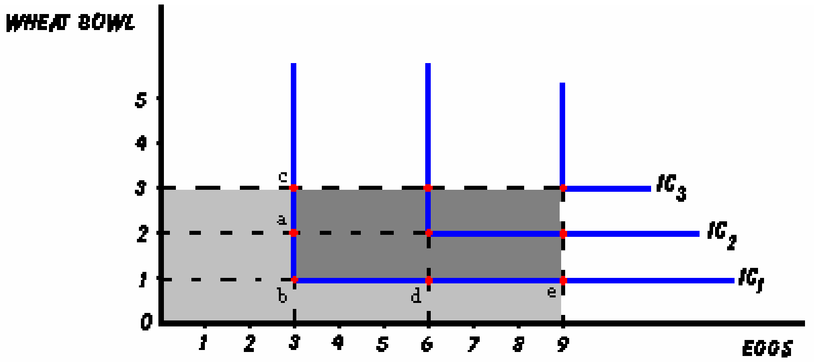
Special case of Interior Solution (other than convex IC)

When commodities are consumed in fixed proportion, Goods are used in a certain proportion.

Example: Doctor's prescription, a patient, when doctor gives a person medicine to take in fixed tablets and capsules, it means that man has maintain the ratio of tablets until he will fine.

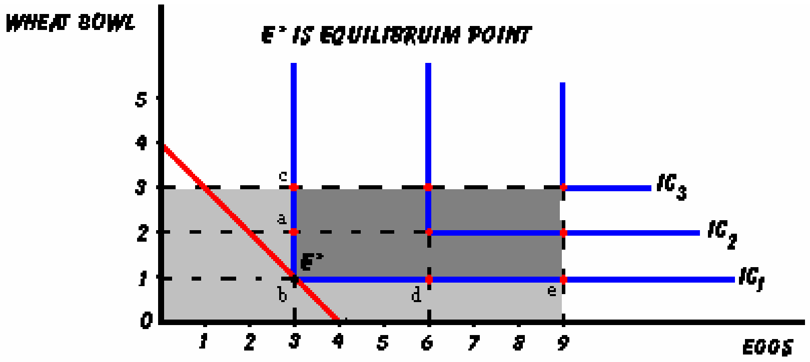
Making cake, a certain proportion of ingredients required, if that required ratio do not maintain then we are unable to make the cake.
If 3 eggs + 1 bowl of wheat is required to make cake then for making one 3:1 ratio is required to make cake, if two then 6:2 with same proportion

No. of Cakes	Egg + Wheat bowl
1	3:1
2	6:2
3	9:3



- If we are at point "a" one extra unit of wheat which can not give utility. Zero utility, those which have one feasible kink point which have more units
- Moving along vertical axis means spending on wheat
- Moving along horizontal direction means spending on equilibrium

3E = 1W
6E = 2W
9E = 3W
12E = 4W



Where commodities are complimentary and used in a fixed proportion

Comparative Static analysis of Consumer Equilibrium

There are three effects of consumer equilibrium

1- Income Effect ⎤ Directly effect budget line
2- Price Effect ⎦
3- Preferences Effect ⎤ This will effect IC map

Income Effect (IE): we have to focus on one commodity, income of consumer increases, what will be the effect on one commodity. There will be effect on consumer equilibrium point due to change in his income, when other factors (Px_1, Px_2, Preferences) remain constant.

Note: We shall only focus on X_1 commodity assuming X_2 is normal commodity.

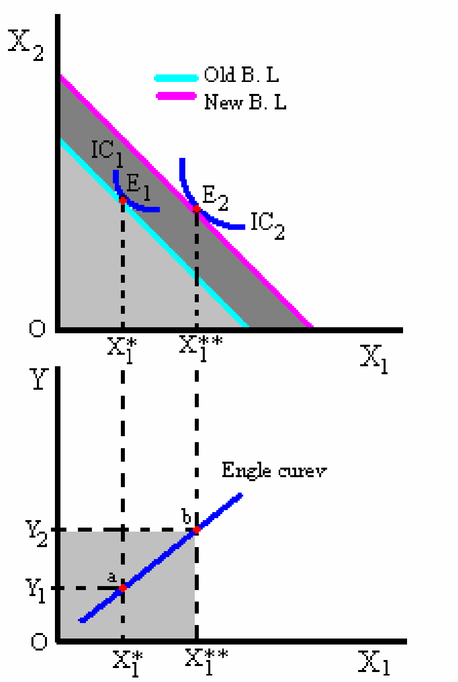
Case1: When income of consumer increases and X_1 is normal commodity

Y increases = X_1 normal commodity

Let assume that we have budget line and equilibrium point

Initial conditions

A_1B_1 = Budget line
E_1 = Equilibrium point
IC_1 = Indifference curve
OX_1* = consumption level of X_1 commodity



Final conditions
A_2B_2 = Budget line
E_2 = Equilibrium point
IC_2 = Indifference curve
OX_1** = consumption level of X_1 commodity

Income Effect
IE = movement from E_1 to E_2
IE = OX_1* to OX_1** (**increases**)
IE = X_1* X_1** (**increases**)

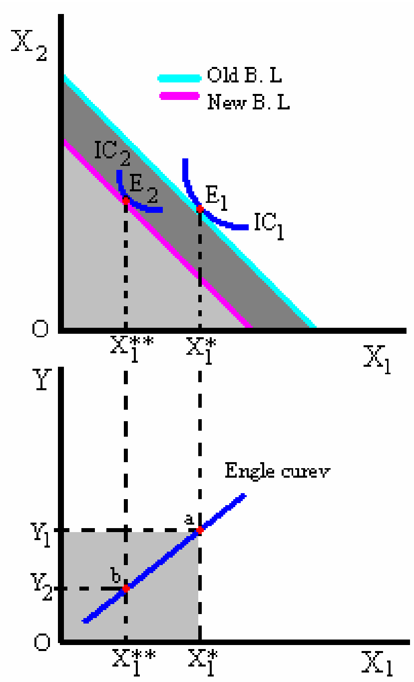
Case 2: When income of consumer decreases and X_1 is normal commodity

"Y" decreases = X_1 normal commodity

Let assume that we have budget line and equilibrium point

Initial conditions
A_1B_1 = Budget line
E_1 = Equilibrium point
IC_1 = Indifference curve

OX_1^* = consumption level of X_1 commodity



<u>Final conditions</u>

A_2B_2 = Budget line

E_2 = Equilibrium point

IC_2 = Indifference curve

OX_1^{**} = consumption level of X_1 commodity

Income Effect

IE = movement from E_1 to E_2

IE = OX_1^* to OX_1^{**} (**decreases**)

IE = $X_1^* X_1^{**}$ (**decreases**)

<u>Case 3:</u> When income of consumer increases and X_1 is inferior commodity

"Y" increases = X_1 inferior commodity

Let assume that we have budget line and equilibrium point

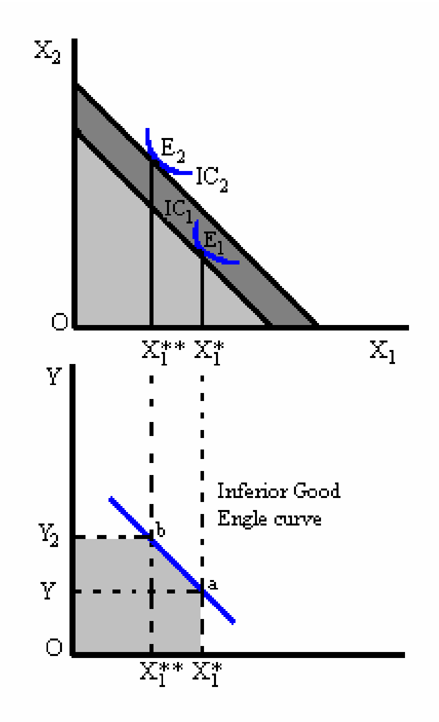
<u>Initial conditions</u>

A_1B_1 = Budget line

E_1 = Equilibrium point

IC_1 = Indifference curve

OX_1^* = consumption level of X_1 commodity



Final conditions

A_2B_2 = Budget line

E_2 = Equilibrium point

IC_2 = Indifference curve

OX_1^{**} = consumption level of X_1 commodity

Income Effect

IE = movement from E_1 to E_2

IE = OX_1^* to OX_1^{**} (**decreases**)

IE = X_1^* X_1^{**} (**decreases**)

Case 4: When income of consumer decreases and X_1 is inferior commodity

"Y" decreases = X_1 inferior commodity

Let assume that we have budget line and equilibrium point

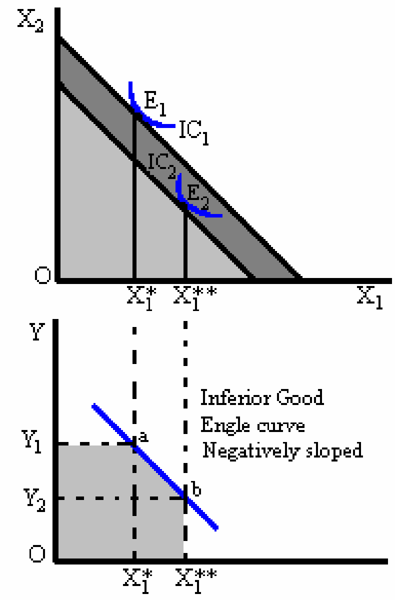
Initial conditions

A_1B_1 = Budget line

E_1 = Equilibrium point

IC_1 = Indifference curve

OX_1^* = consumption level of X_1 commodity



<u>Final conditions</u>

A_2B_2 = Budget line

E_2 = Equilibrium point

IC_2 = Indifference curve

OX_1^{**} = consumption level of X_1 commodity

Income Effect

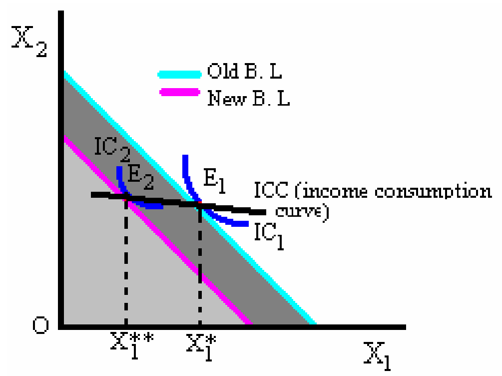
IE = movement from E_1 to E_2

IE = OX_1^* to OX_1^{**} (**increases**)

IE = X_1^* X_1^{**} (**increases**)

ICC (Income consumption curve)

When we join the equilibrium points by a line is known as ICC.



Price Effect (PE):

Effect on consumer equilibrium point due to change in price of one commodity with all other factors held constant.

Let commodities are X_1 and X_2 and other factors are held constant

Case 1: P_{x1} increases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 normal commodity

Let assume that we have budget line and equilibrium point

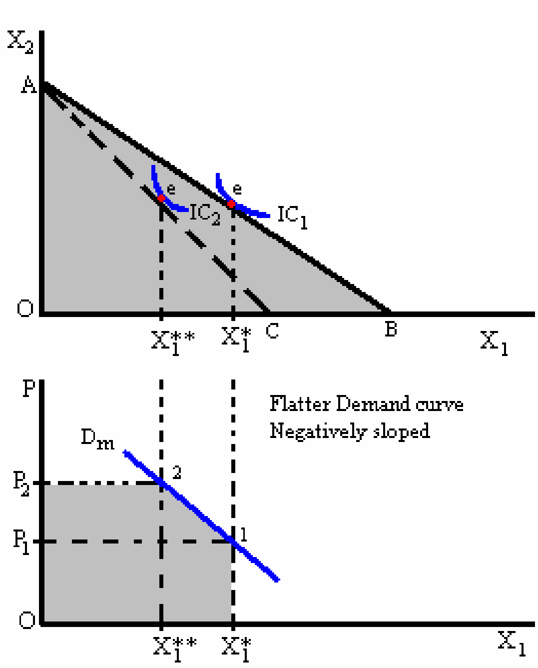
Initial conditions

A_1B_1 = Budget line

E_1 = Equilibrium point

IC_1 = Indifference curve

OX_1^* = consumption level of X_1 commodity



Final conditions

A_2B_2 = Budget line

E_2 = Equilibrium point

IC_2 = Indifference curve

OX_2^* = consumption level of X_1 commodity

Price Effect

PE = movement from E_1 to E_2

PE = OX_1^* to OX_1^{**} (**decreases**)

PE = X_1^* X_1^{**} (**decreases**)

Case 2: P_{x1} decreases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 normal commodity

Let assume that we have budget line and equilibrium point

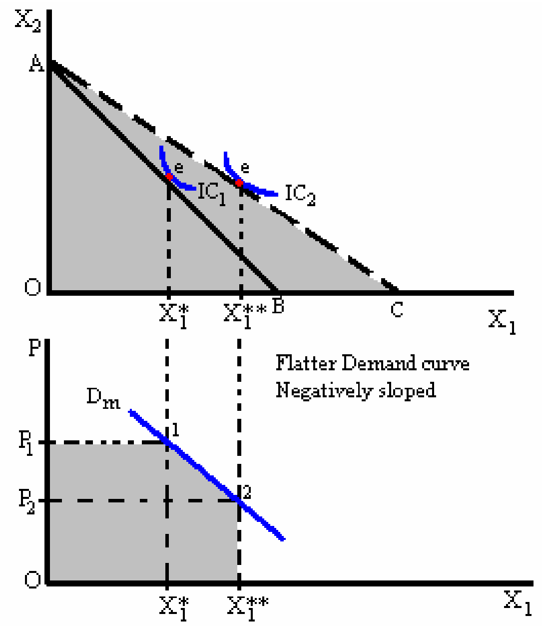
Initial conditions

A_1B_1 = Budget line

E_1 = Equilibrium point

IC_1 = Indifference curve

OX_1^* = consumption level of X_1 commodity



<u>Final conditions</u>

A_2B_2 = Budget line

E_2 = Equilibrium point

IC_2 = Indifference curve

OX_2^* = consumption level of X_1 commodity

Price Effect

PE = movement from E_1 to E_2

PE = OX_1^* to OX_1^{**} (**increases**)

PE = X_1^* X_1^{**} (**increases**)

<u>Case 3:</u> Px_1 increases, $\bar{Y}$, $\overline{Px_2}$, $\overline{Pref}$, X_1 inferior commodity

Let assume that we have budget line and equilibrium point

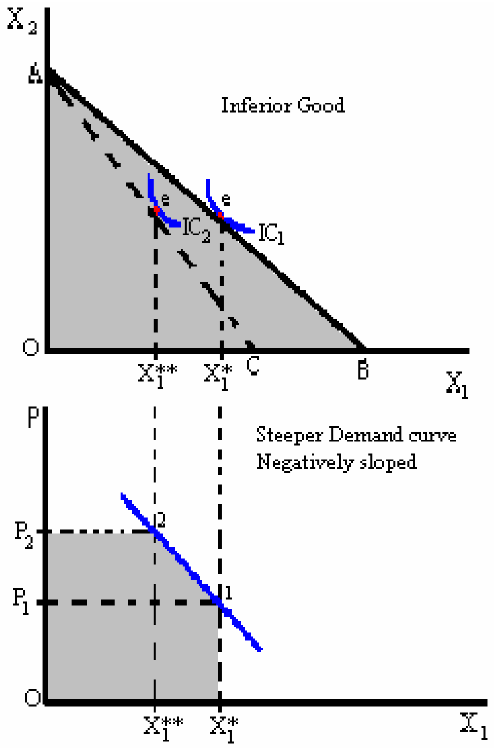
<u>Initial conditions</u>

A_1B_1 = Budget line

E_1 = Equilibrium point

IC_1 = Indifference curve

OX_1^* = consumption level of X_1 commodity



<u>Final conditions</u>

A_2B_2 = Budget line

E_2 = Equilibrium point

IC_2 = Indifference curve

OX_2* = consumption level of X_1 commodity

Price Effect

PE = movement from E_1 to E_2

PE = OX_1* to OX_1** (**decreases**)

PE = $X_1* X_1**$ (**decreases**)

<u>Case 4:</u> P_{x1} decreases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 inferior commodity

Let assume that we have budget line and equilibrium point

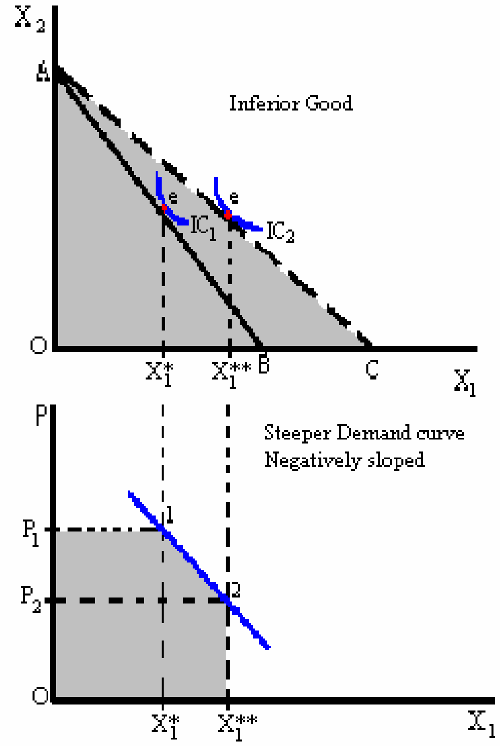
<u>Initial conditions</u>

A_1B_1 = Budget line

E_1 = Equilibrium point

IC_1 = Indifference curve

OX_1* = consumption level of X_1 commodity



Final conditions

A_2B_2 = Budget line

E_2 = Equilibrium point

IC_2 = Indifference curve

OX_2* = consumption level of X_1 commodity

Price Effect

PE = movement from E_1 to E_2

PE = OX_1* to OX_1** (**increases**)

PE = X_1* X_1** (**increases**)

Case 5: P_{x1} increases, $\bar{Y}$, $\bar{P}_{x2}$, $\overline{Pref}$, X_1 Giffen commodity

Let assume that we have budget line and equilibrium point

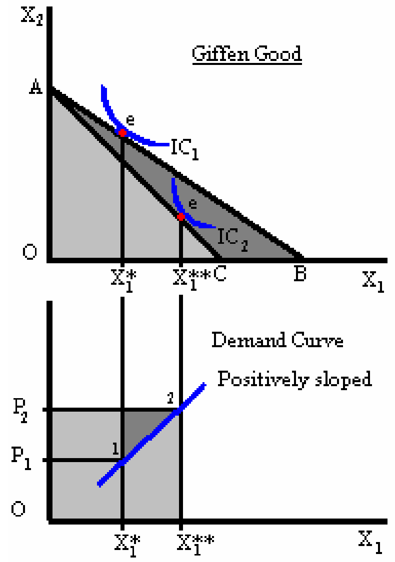
Initial conditions

A_1B_1 = Budget line

E_1 = Equilibrium point

IC_1 = Indifference curve

OX_1* = consumption level of X_1 commodity



Final conditions

A_2B_2 = Budget line

E_2 = Equilibrium point

IC_2 = Indifference curve

OX_2* = consumption level of X_1 commodity

Price Effect

PE = movement from E_1 to E_2

PE = OX_1* to OX_1** (**increases**)

PE = $X_1* X_1**$ (**increases**)

Case 6: P_{x1} decreases, Y, P_{x2}, Pref, X_1 Giffen commodity

Let assume that we have budget line and equilibrium point

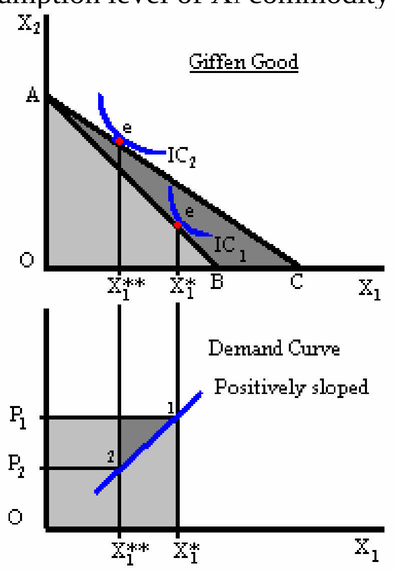
Initial conditions

A_1B_1 = Budget line

E_1 = Equilibrium point

IC_1 = Indifference curve

OX_1* = consumption level of X_1 commodity



<u>Final conditions</u>

A_2B_2 = Budget line

E_2 = Equilibrium point

IC_2 = Indifference curve

$OX_2{}^*$ = consumption level of X_1 commodity

Price Effect

PE = movement from E_1 to E_2

PE = $OX_1{}^*$ to $OX_1{}^{**}$ (**decreases**)

PE = $X_1{}^*$ $X_1{}^{**}$ (**decreases**)

Commodities	Price Effect	Relationship
Normal	PE = $X_1{}^*$ $X_1{}^{**}$ decreases if Px_1 increases	**Strong negative**
Inferior	PE = $X_1{}^*$ $X_1{}^{**}$ decreases if Px_1 increases	**Weak negative**
Giffen	PE = $X_1{}^*$ $X_1{}^{**}$ increases if Px_1 increases	**Positive**

<u>Decomposition of Price effect into income effect and substitution effect</u>

Let

Px_1 increases {
- **Purchasing Power (real income) decreases**
- **X_1 is now relatively expensive as compare to X_2.**

(Basic principle of analysis commodities is substitutes)

Purchasing power means change in demand due to change in real income of consumer. It is **"income effect"**.

Demand for X_1 decreases because of X_2 substitute for X_1 by some amount. It is "**substitution effect"**.

Price Effect = Substitution Effect + Income Effect

P$_x$ increases ⇔ **Real income** decreases ⇔ **D**$_x$ decreases

Income effect will be negative (-)

P$_x$ increases ⇔ **relatively** expensive as compare to X_2 ⇔ **D**$_x$ decreases

Substitution effect will be negative (-)

Price effect = Income effect + Substitution effect

(-) = (-) + (-)

Income Effect, Positive Relationship [+ shows Normal Good]

P$_x$ increases ⇔ **Real income** decreases ⇔ **D**$_x$ decreases

Substitution Effect, Negative Relationship

- Normal commodity has negative (-) relationship with price and positive (+) with real income

Income Effect, Negative Relationship [- shows Inferior Good]

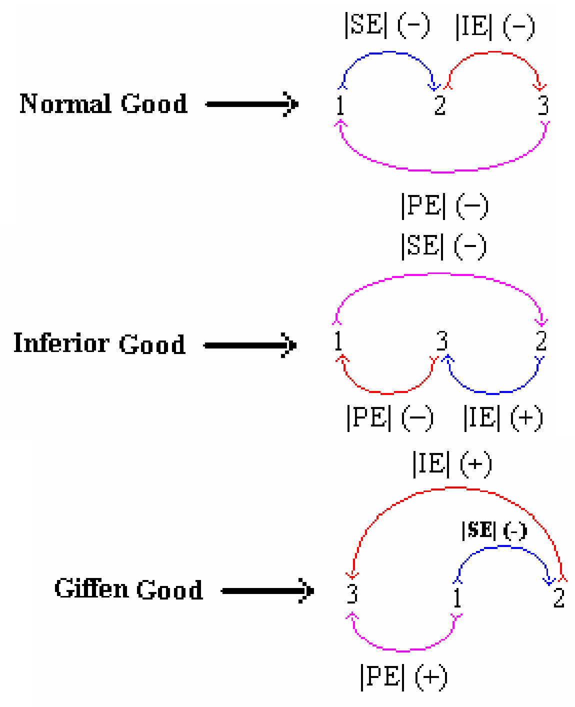
Px increases ⇔ **Real income** decreases ⇔ **D**x decreases

Substitution Effect, Positive Relationship

- Inferior commodity has positive (+) relationship with price and negative (-) with income

Income effect + Substitution effect = Price effect

Normal Good	**(-)**	+	**(-)**	=	**(-)**
Inferior Good	**(+)**	+	**(-)**	=	**|IE| > |SE| (+) Giffen** **|IE| <|SE| (-) Inferior**



- In order to decompose total price effect into income effect and substitution effect, real income (purchasing power) of the consumer is held constant then the effect would be only substitution effect, finally SE will be compared with total price effect to work out IE.

- To keep real income constant there are two approaches
  - Hicksian Approach
  - Slutsky Approach

- **Hicksian Approach**

To keep real income constant, consumer is compensated in such a way that he enjoys the previous level of satisfaction, i.e. he gets back on old IC, “Satisfaction level is held constant”.

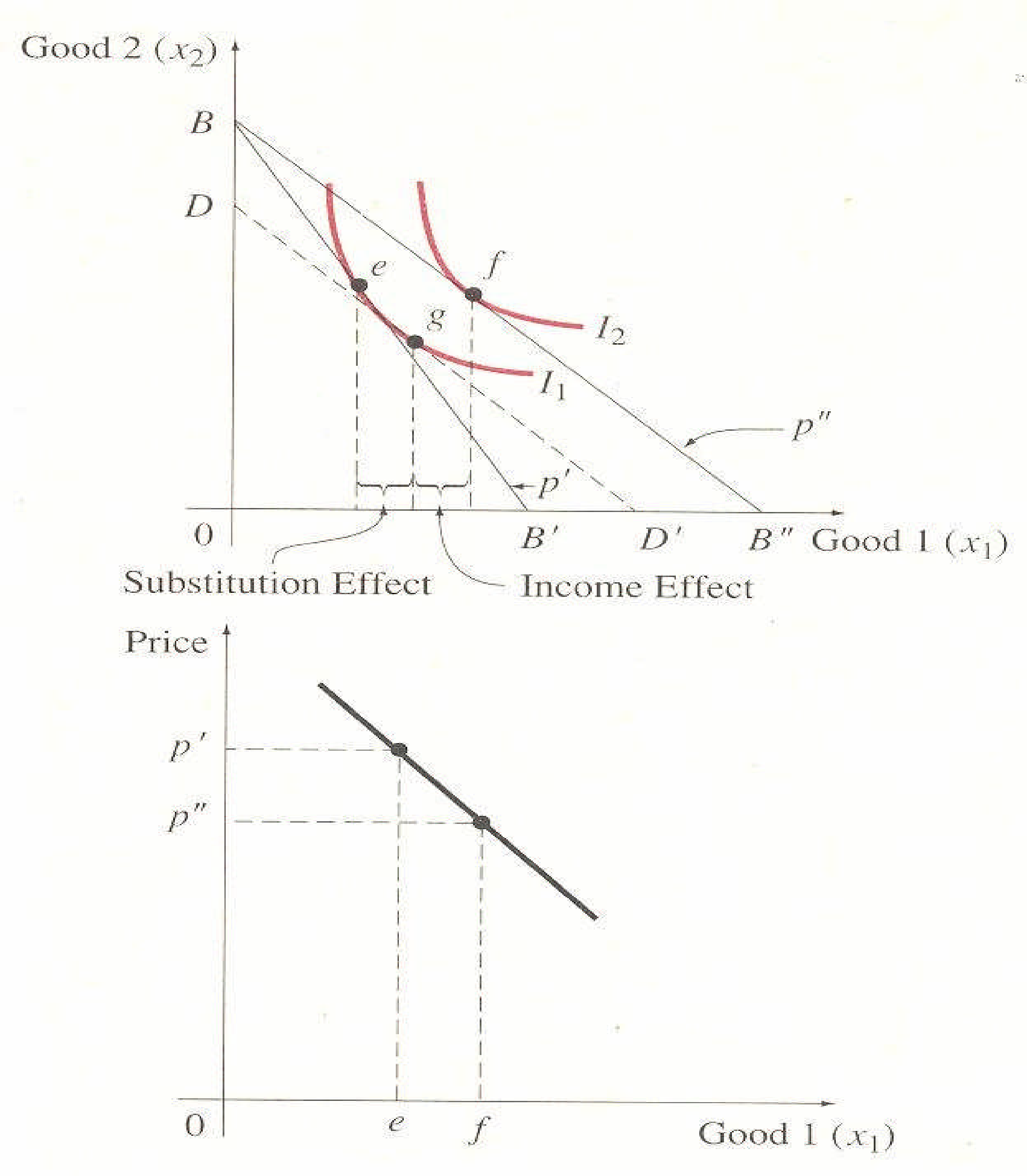
Graphically: A hypothetical budget line is drawn which is parallel to new budget line and it is tangent to old (initial) IC.

There are six cases of it.

<u>Case 1:</u> P_{x1} increases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 normal commodity

Price Effect	=	**Substitution Effect**	+	**Income Effect**
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

Case 2: P_{x1} decreases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 normal commodity



Price Effect	=	**Substitution Effect**	+	**Income Effect**
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

Case 3: P_{x1} increases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 inferior commodity

Price Effect	=	**Substitution Effect**	+	**Income Effect**
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

<u>Case 4:</u> **P_{x1} decreases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 inferior commodity**

Price Effect	=	Substitution Effect	+	Income Effect
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

Case 5: P_{x1} increases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 Giffen commodity

Price Effect	=	**Substitution Effect**	+	**Income Effect**
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

Case 6: P_{x1} **decreases,** $\overline{Y}$**,** $\overline{P_{x2}}$**,** $\overline{Pref}$**,** X_1 **Giffen commodity**

Price Effect	=	**Substitution Effect**	+	**Income Effect**
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

- SE is independent of all commodities nature and same for all commodities that are negative.
- D_M : Income effect on the demand for a good in response to change in its own price
  Also ordinary demand curves
  $D_M \Rightarrow \Delta P \Rightarrow \Delta D$
  $D_M = f$ (Prices, Income)
  $D_M = f(P_1, P_2, Y)$
- D_H = which shows substitution effect on the demand for a good in response to change in its own price against the given price of the goods, along D_M utility remains constant.
  $D_H = f$ (Prices, Utility)
  $D_H = f((P_1, P_2, U)$

Utility held constant curves/functions, ordinary or **Marshallian** demand curve shows **income effect** and **Hicksian** shows **substitution effect**. Hicksian demand curve is compensatory demand curve.

- **Slutsky Approach**

To keep real income constant, consumer is compensated in such a way that he goes back on to initial consumption level, i.e. he gets back on old consumption bundle.

Graphically: A hypothetical budget line is drawn which is parallel to new budget line and crossing through old budget line from initial equilibrium point.

There are six cases of it.

<u>Case 1:</u> P_{x1} increases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 normal commodity

- IC should be in between the hypothetical budget line and old budget line

Price Effect	=	Substitution Effect	+	Income Effect
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

Case 2: P_{x1} **decreases,** $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 **normal commodity**

Price Effect	=	Substitution Effect	+	Income Effect
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

<u>Case 3:</u> P_{X1} increases, $\overline{Y}$, $\overline{P_{X2}}$, $\overline{Pref}$, X_1 inferior commodity

Price Effect	=	**Substitution Effect**	+	**Income Effect**
(E_A to E_C)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

<u>Case 4:</u> P_{x1} decreases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 inferior commodity

Price Effect	=	**Substitution Effect**	+	**Income Effect**
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

Case 5: P_{x1} increases, $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 Giffen commodity

Price Effect	=	**Substitution Effect**	+	**Income Effect**
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

Case 6: P_{x1} **decreases,** $\overline{Y}$, $\overline{P_{x2}}$, $\overline{Pref}$, X_1 **Giffen commodity**

Price Effect	=	**Substitution Effect**	+	**Income Effect**
(E_a to E_c)	=	(E_a to E_b)	+	(E_b to E_c)
(X_{1a} to X_{1c}) decreases	=	(X_{1a} to X_{1b}) decreases	+	(X_{1b} to X_{1c}) decreases
(–)	=	(–)	+	(–)

Theory of Cost and Production:

Cost { Cost equation, Cost function

Production { Production equation, Production function

We start from Cost equation

Cost equation:

$$C = \sum \omega_i .X_i$$

(Where i = 1, 2, 3… n)

ω_i = Price of ith input
X_i = Quantity of input

Therefore

$$C = (\overline{w}).L + (\overline{r}).\overline{K})$$ (if firm has only two inputs)

Cost equation gives us total cost of production. It is simple product of W and X.

Cost function:

$C = f$ (Input prices, output)
$C = f(\omega_i, Y)$. Where (ω_i is given for a producer or firm)

We have two types for analysis, which are given below;

Short-Period:

Short period is one in which full adjustment is not possible, if one factor is fixed then we can say that it is short period, during which consumers and producers have not had enough time to make all the adjustments to the new situation.

Long-Period:

Long period is one during which consumers and producers have had enough time to make all the adjustments to a new situation, all factors are variables.

Short-Period Analysis

As we know about short-period analysis for the cost of production, therefore we start with cost equation.

Cost equation in short-period

$$C = \underbrace{(\bar{w}).L}_{\substack{\text{Variable} \\ \text{(Labor cost)}}} + \underbrace{(\bar{r}).\bar{K}}_{\substack{\text{fixed} \\ \text{(Capital cost)}}}$$

Forms of cost:

1- **Total cost (TC)**

The total expenditure put by a firm to produce a certain amount of a good is called as total cost (TC) of production.

Mathematically:

$$TC = TVC + TFC$$

2- **Total fixed cost (TFC)**

In short-period analysis, the amount of capital is fixed, so the cost on the capital is fixed. It is independent of output.

3- **Total variable cost (TVC)**

It is the cost born by a firm for the production of output by employing variable factors (labor).

4- **Average cost (AC)**

It is the ratio of costs over output produced.

Mathematically we can measure these average costs as:

i- Average total cost

$$ATC = \frac{TC}{Q}$$

ii- Average variable cost

$$AVC = \frac{TVC}{Q}$$

iii- Average fixed cost

$$AFC = \frac{TFC}{Q}$$

5- Marginal cost (MC)

It is the change in total cost due to additional one unit change in output.

Mathematically:

$$MC = \frac{\Delta TC}{\Delta Q_{=1}}$$

Or

$$MC = \frac{\Delta TVC}{\Delta Q_{=1}}$$

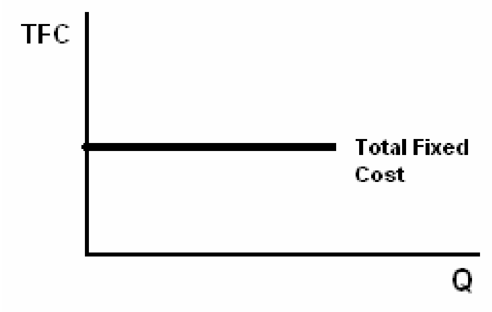
Behavior of total cost, total fixed cost and total variable cost, average costs and marginal costs as well:

Q	TFC	TVC	TC	MC	AFC	AVC	ATC
0	10	0	10	-	-	-	-
1	10	28	38	28	10	28	38
2	10	54	64	26	5	27	32
3	10	75	85	21	3.33	25	28.33
4	10	95	105	20	2.50	23.90	26.25
5	10	126	136	31	2	21.20	27.20
6	10	168	178	42	1.67	2.8	29.67
7	10	214	224	46	1.42	30.50	32
8	10	264	274	50	1.25	33	34.25
9	10	320	330	56	1.11	35.55	36.37
10	10	390	400	70	1	39	40

Graphically:

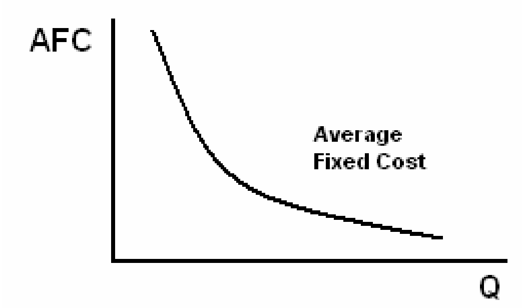
Total Fixed Cost:

Total fixed cost is parallel to horizontal axes, so its slope is always zero, if we take it on the horizontal axes.

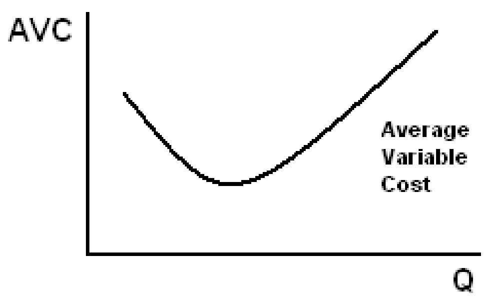


Average Fixed Cost:

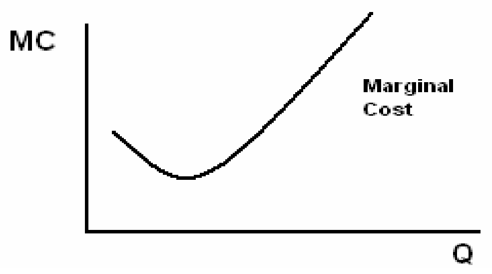
This curve does not touch the axes.



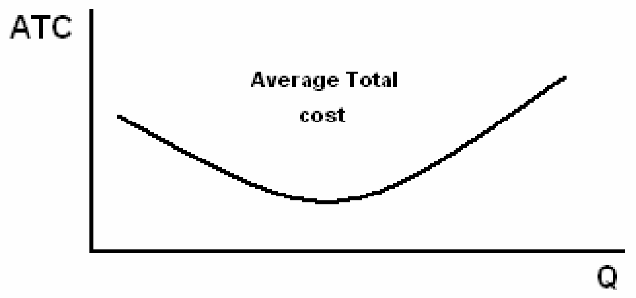
Average Variable Cost:



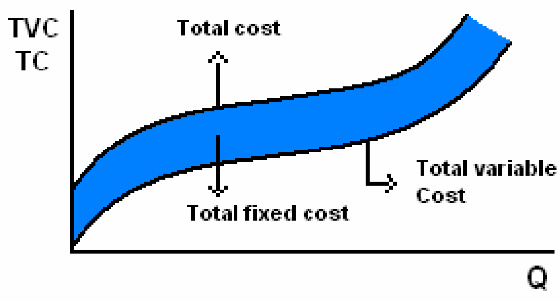
Marginal Cost:



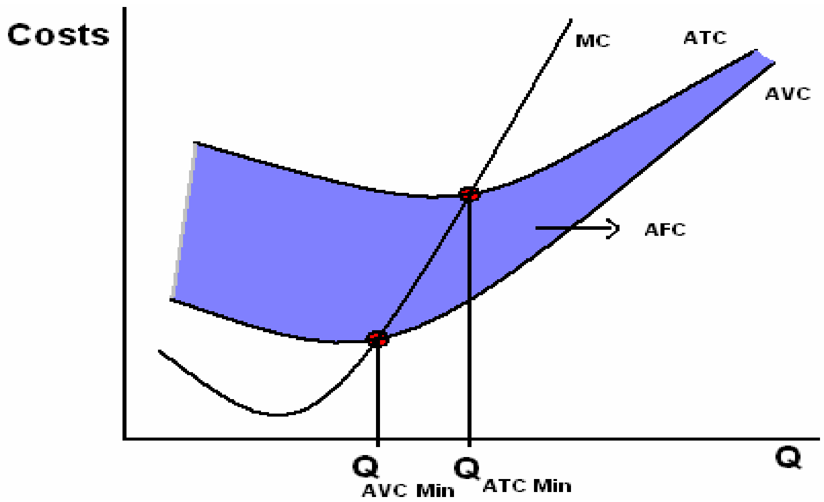
Average Variable Cost:



Relationship between TVC and TC:



Relationship between AVC, ATC and MC:

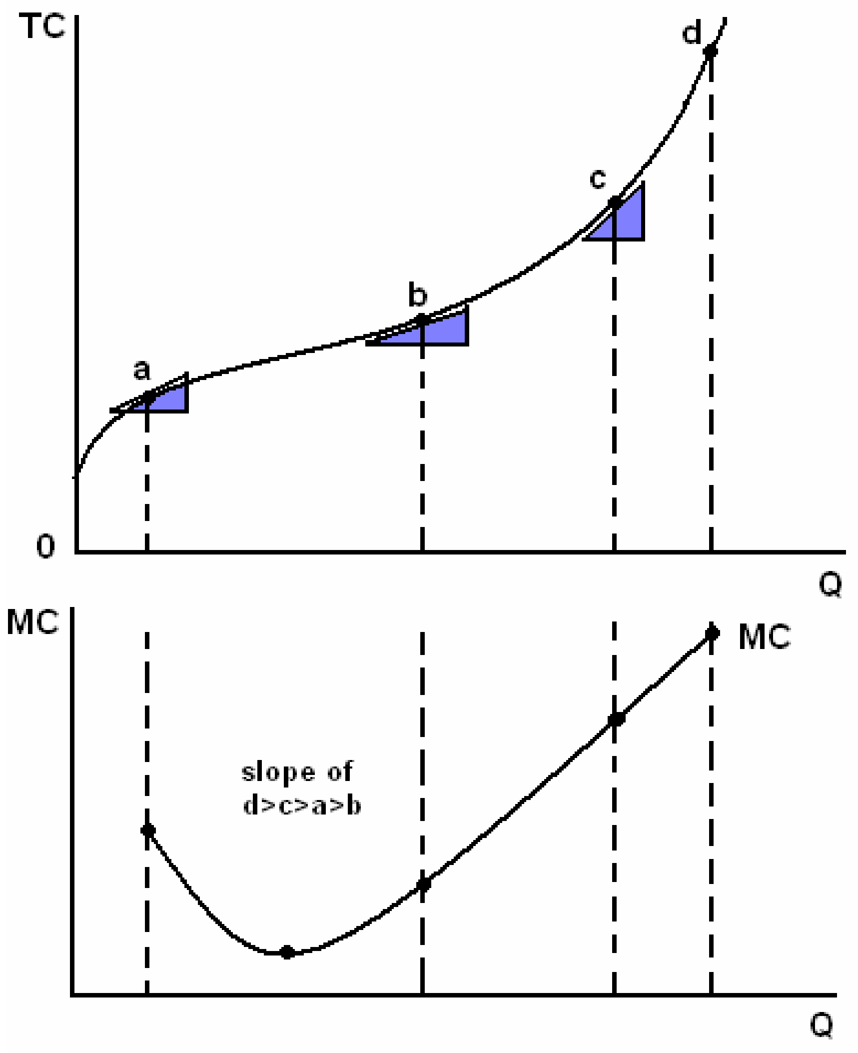


- Gap between ATC and AVC is AFC which is continuously declining but remains positive, ATC > AVC.
- Minimum of AVC comes earlier than the minimum of ATC.

- MC > ATC and AVC initially, but finally MC > ATC and AVC.
- MC cuts ATC and AVC at there minimum points.
- MC's minimum point is before ATC and AVC.

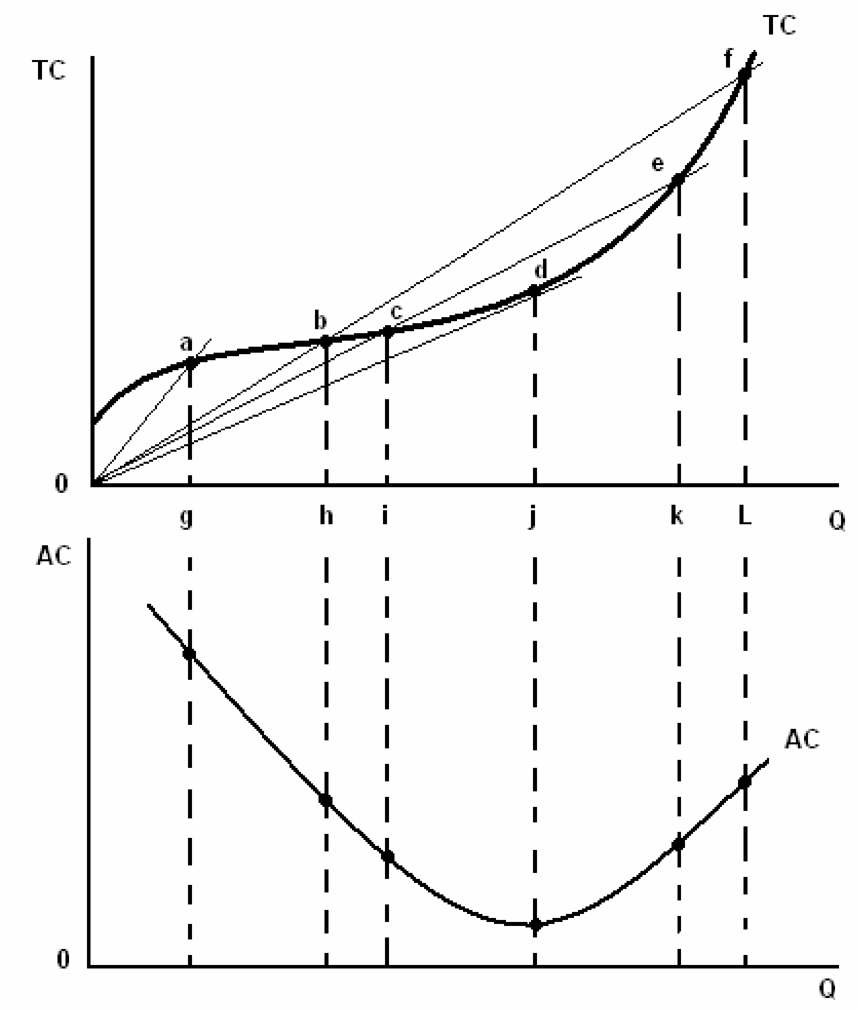
Derivation of AC and MC curves from TC curve:

1. Slope of TC is MC



As TVC is parallel to TC, so we can also derive MC from TVC.

2. Slope of rays from origin to TC is AC.



Slope at point "a" $= \frac{ag}{og} > slope\ at\ point\ b = \frac{bh}{oh}$

Production in Short–Period

Productions function in short-period:

$$Q = f(\bar{K}, L)$$

- To change Q (ΔQ), we have to change L (ΔL)

Or

- ΔL => ΔQ with fixed amount of capital.

Form of Production:

There are three forms of production;

i- Total Product of Labor (TP_L).

ii- Average Product of Labor (AP_L).

iii- Marginal Product of Labor (MP_L).

Total Product of Labor (TP_L):

The sum of all units produced by all units of labor is called total product of labor.

Average Product of Labor (AP_L):

It is the per labor output

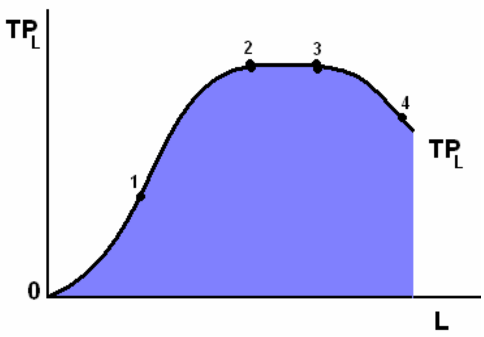
$$AP_L = \frac{TP_L}{L}$$

Marginal Product of Labor (MP_L):

It is the change of total product of labor due to additional unit of labor, if we change labor by a constant amount, output changes. But change in output is not fixed. Output changes with variable amount.

Mathematically:

$$MP_L = \frac{\Delta TP_L}{\Delta L_{=1}}$$



Law of Variable Production:

According to the ***law of variable proportion***, "output initially increases with increasing rate, then increases at a constant rate and after that it increases at decreasing rate".

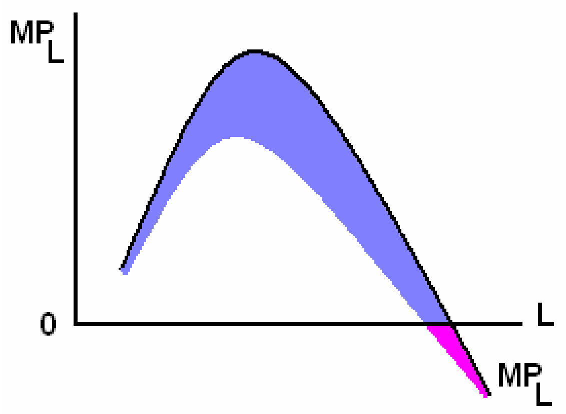
In the diagram: There are five phases;

1st Phase: TP_L increases at increasing rate (0 →1).
2nd Phase: TP_L increases at decreasing rate (1→2).
3rd Phase: TP_L increases at constant rate (2 →3).
4th Phase: TP_L gets at maximum point (stop increasing).
5th Phase: TP_L decreases at decreasing rate (3 →4).

Behavior of TP_L, MP_L and AP_L:

L	*TP_L*	*MP_L*	*AP_L*
0	0	--	--
1	2	2	2
2	5	3	2.5
3	12	7	4
4	24	12	6
5	28	4	5.6
6	30	2	5
7	30	0	4.22
8	28	-2	3.5
9	24	-4	2.67
10	18	-6	1.8

MP_L:



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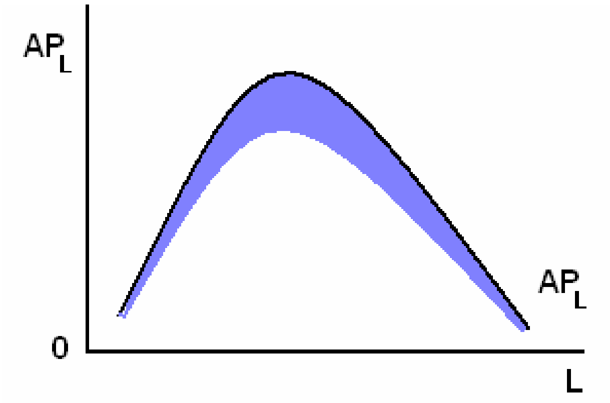
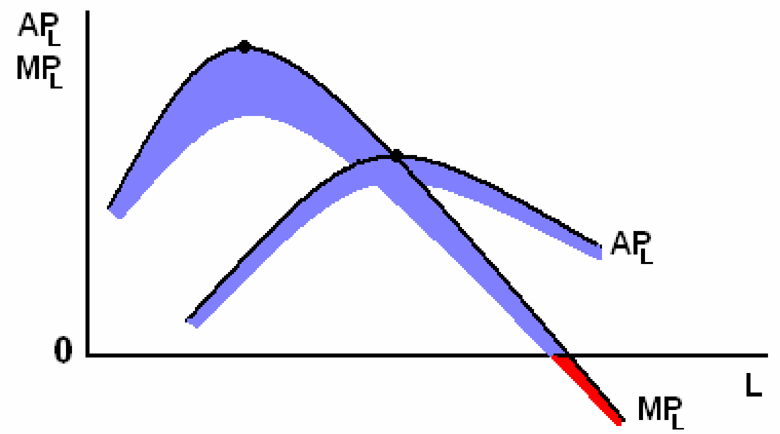
MP_L initially increases, gets maximum then starts decreasing, and gets zero and then become negative.

AP_L:



AP_L has same behavior as **MP_L** but remains positive.

Relationship between MP_L and AP_L:



- When MP_L is increasing, $MP_L > AP_L$.
- When MP_L is maximum, $MP_L = AP_L$.
- When MP_L is decreasing, $MP_L < AP_L$.
- Maximum of MP_L comes before the maximum of AP_L.

$$AP_L = \frac{TP_L}{L}$$

$$TP_L = AP_L . L$$

$$\frac{dTP_L}{dL} = AP_L.(1) + L.\frac{dAP_L}{dL}$$

$$MP_L = AP_L + L(Slope\ of\ AP_L) \longrightarrow A$$

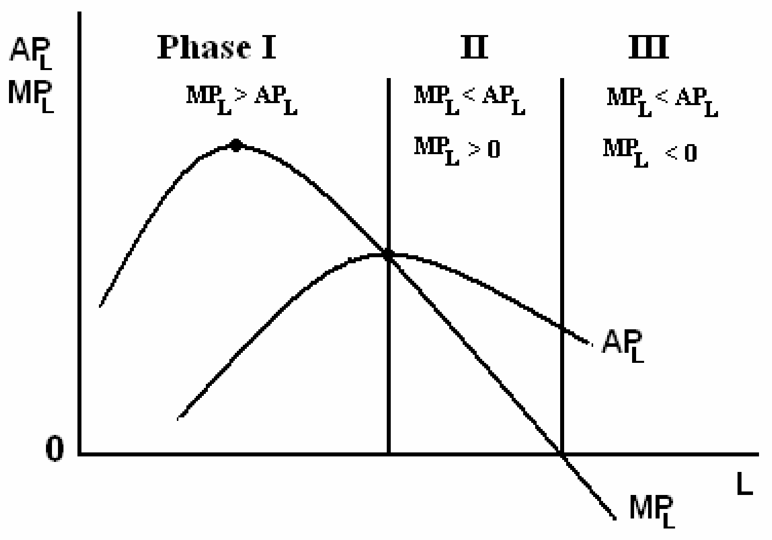
Expression "A" tells us that:

- As slope of $AP_L > 0$, then $AP_L < MP_L$ when AP_L is increasing.
- As slope $AP_L = 0$, then $AP_L = MP_L$ when AP_L is maximum.
- As slope of $AP_L < 0$, then $AP_L > MP_L$ when AP_L is decreasing.

Phases of Production

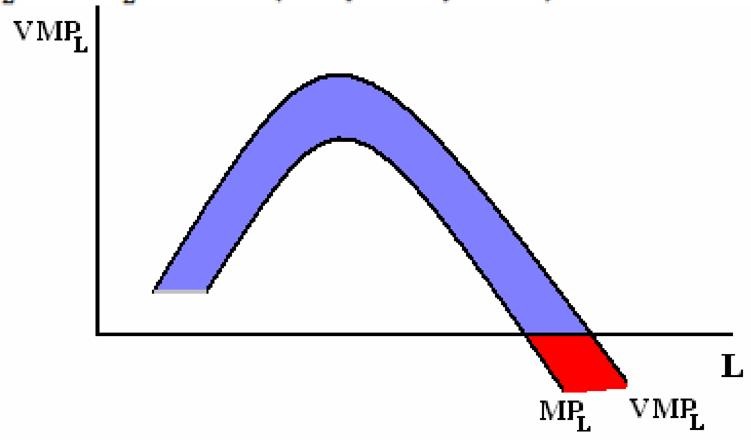
There are three phases of production (as we can see in the graph).

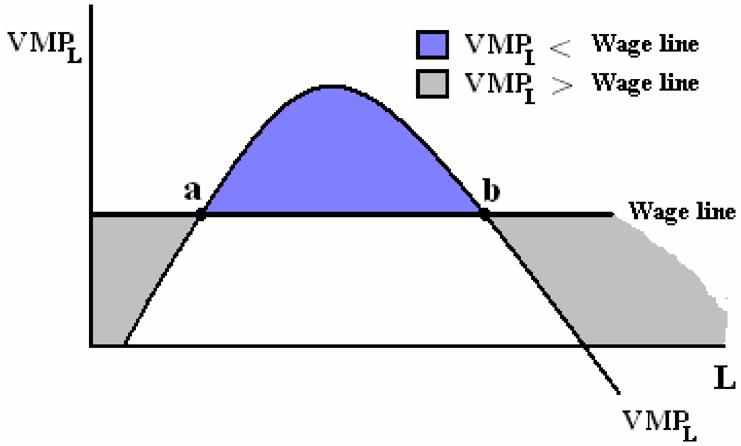
Decision of Employment level: Firm will continue to employ labor as for as the contribution of labor is greater than or equal to his wage provided that VMP_L is decreasing. Labor will be employed as for as $VMP_L \geq$ wage.



MP_L is the contribution of labor, in order to workout the contribution of labor VMP_L is calculated;

$$VMP_L = MP_L . w \qquad (w = price\ of\ labor)$$





- Where “a” and “b” are the break even points (no profit no loss)

Elasticity of Production:

It is the proportionate change in output due to proportionate change in inputs, since labor is the only variable factor of production in short-period analysis, so elasticity of production is the % change in output due to 1% change in labor.

$$\varepsilon_p = \frac{Proportionate\ change\ in\ output}{Proportionate\ change\ in\ labor} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta L}{L}}$$

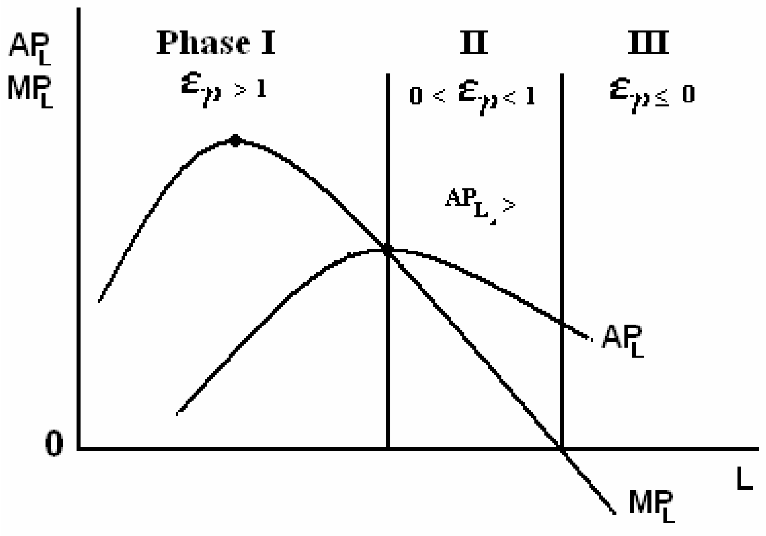
$$\varepsilon_p = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta L}{L}}$$

$$\varepsilon_p = \frac{\Delta Q}{\Delta L}.\frac{L}{Q}$$ (Where Q is TP_L)

As $$MP_L = \frac{\Delta TP_L}{\Delta L} = \frac{\Delta Q}{\Delta L}$$

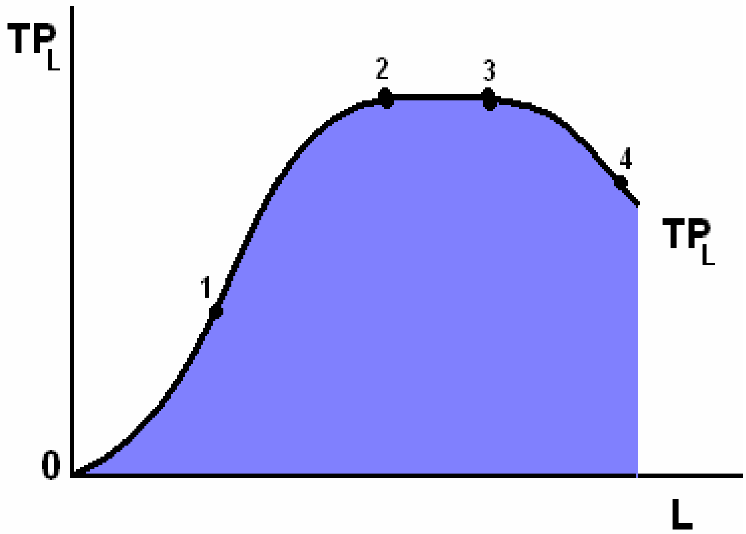
And $$AP_L = \frac{TP_L}{L} = \frac{Q}{L}$$

So $$\varepsilon_p = \frac{MP_L}{AP_L}$$



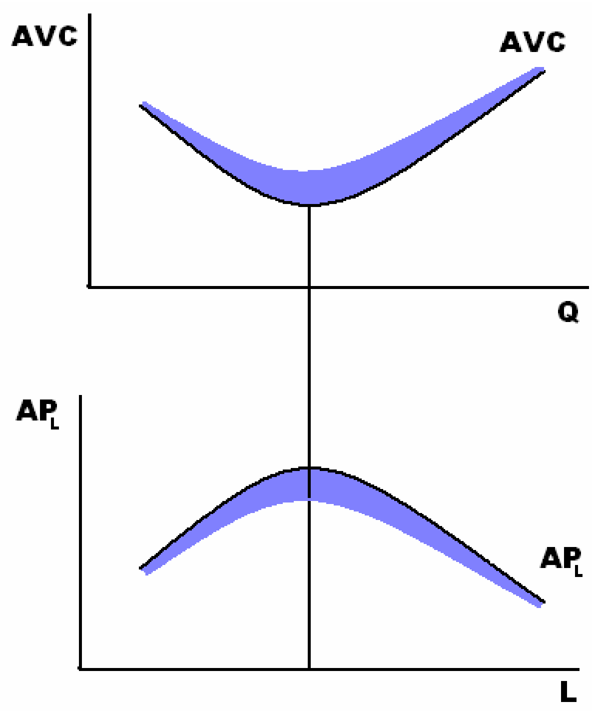
Relationship or Duality between Production and Cost

1- TP_L and TC:



TP_L is increasing at increasing rate (till point 1), it means that every next unit of labor produces more, but every next unit of output has lower cost, it means that total cost of production increases at decreasing rate.

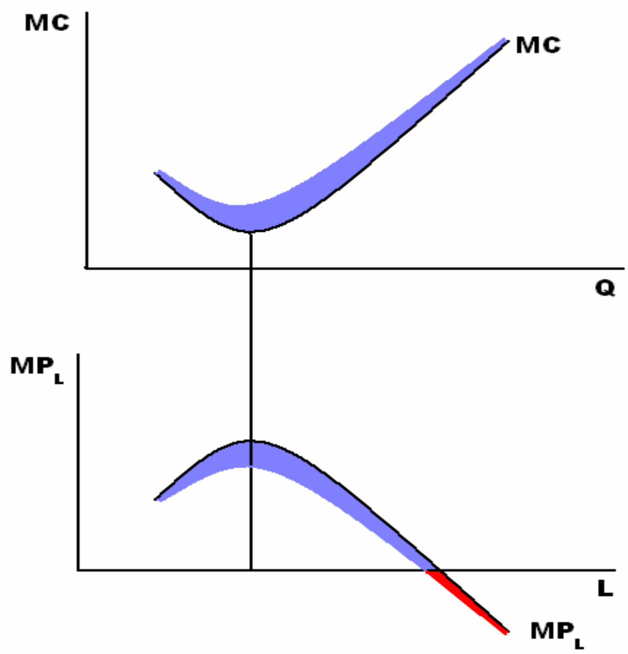
2- AP_L and AVC:



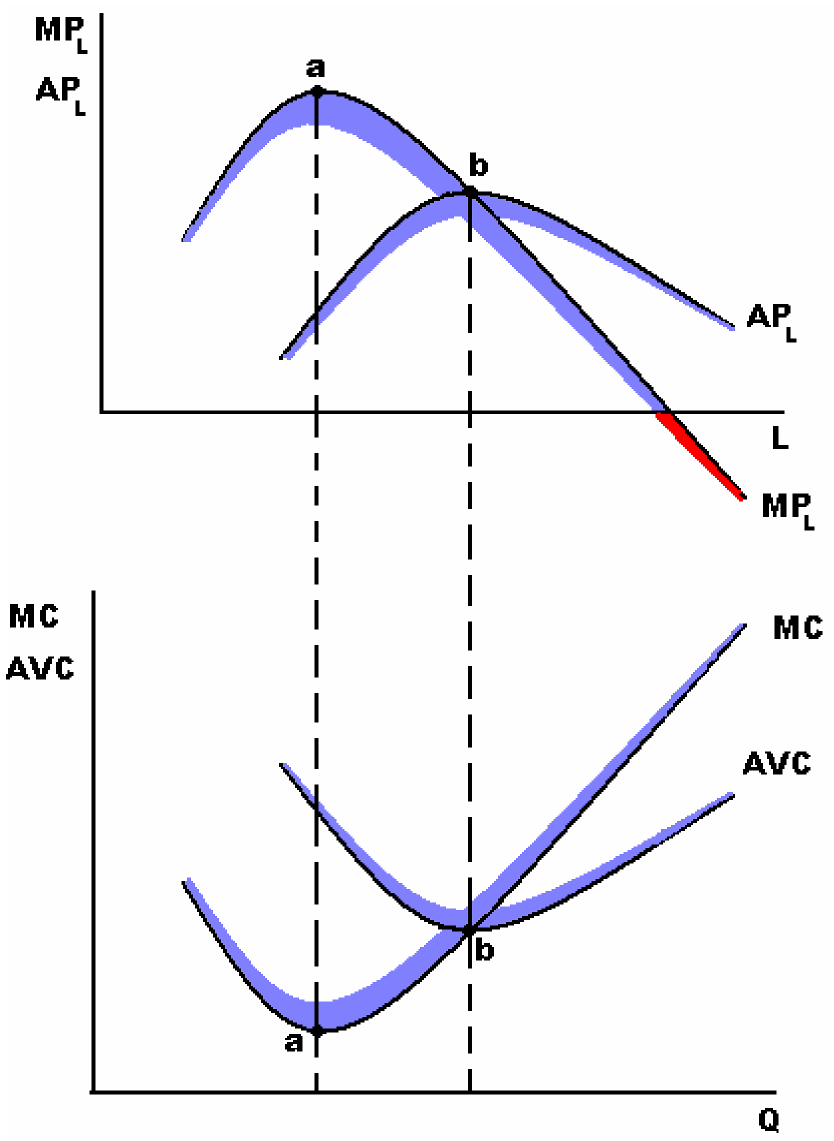
It means that,

- When AVC is decreasing, AP_L is increasing
- When AVC is minimum, AP_L is gets maximum
- When AVC is increasing, AP_L is decreasing, so these are simple mirror images of each other

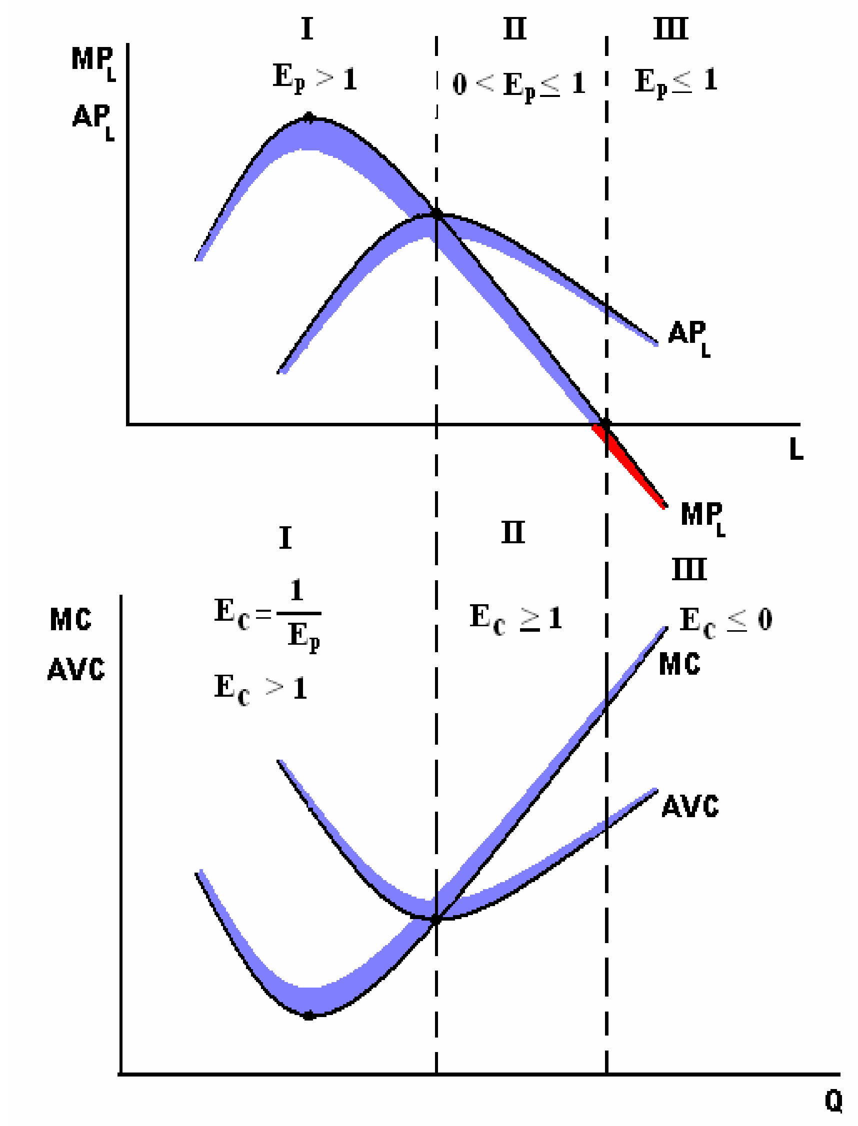
3- MP_L and MC:



Mirror images:



Mirror images and Elasticity's:



Elasticity of cost:

It is the proportionate change in total cost due to proportionate change in output. Since labor is the only factor of production which is variable in short-period, so E_C is the %age (proportionate) change in total cost due to 1% change in labor.

$$\varepsilon_c = \frac{\text{proportionate change in total cost}}{\text{proportionate change in Labor}}$$

$$\varepsilon_c = \frac{\Delta TC/C}{\Delta L/L}$$

$$\varepsilon_c = \frac{\Delta TC}{\Delta L} \cdot \frac{L}{C} \quad (where\ C\ is\ TC)$$

$$AS \quad MC = \frac{\Delta TC}{\Delta L}$$

$$and \quad AVC = \frac{L}{c}$$

$$AS \quad \varepsilon_C = \frac{MC}{AVC}$$

$$AS \quad \varepsilon_p = \frac{AVC}{MC} \qquad so\ \ \varepsilon_C = \frac{1}{\varepsilon_p}$$

Long period Analysis

As we know that long-period is one which is the sum of many short-periods and in long-period both capital and labor are variable.

Cost in Long-period:

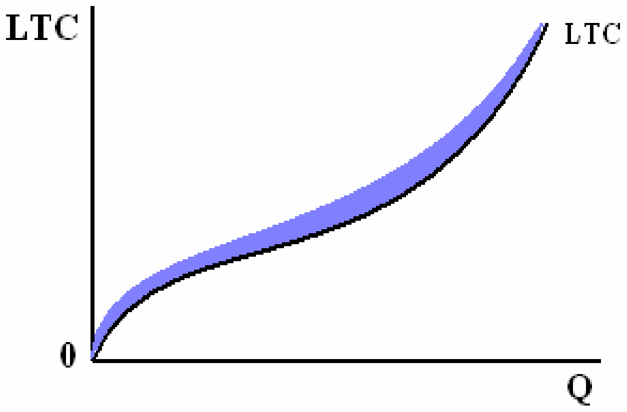
Cost equation

$$C = \underbrace{w.L}_{\substack{\text{Variable.} \\ \text{(Labor cost)}}} + \underbrace{r\ K}_{\substack{\text{Variable} \\ \text{(Capital cost)}}}$$

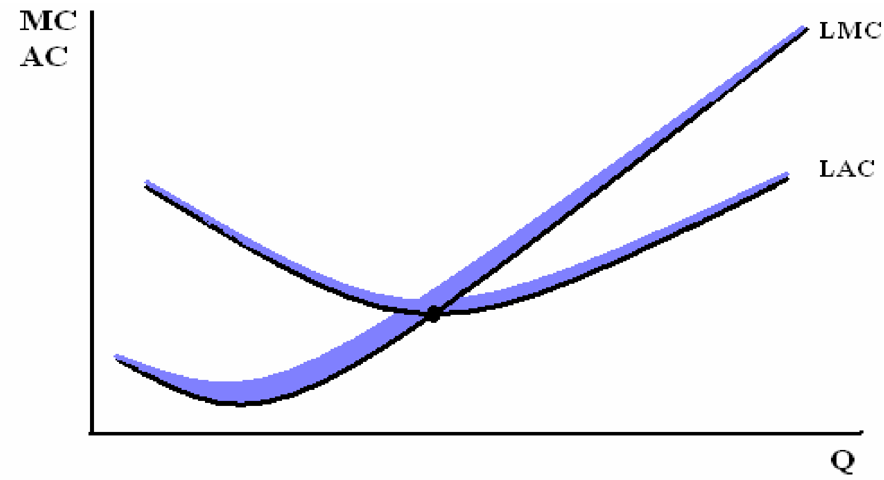
Forms of Cost:

- **i- Long run Total cost (LTC)**
- **ii- Long run Average cost (LAC)**
- **iii- Long run Marginal cost LMC)**

<u>Long-run Total cost curve:</u>



MC and AC curves in long-run:



Return to Scale

It is the increase in output when we increase the amount of input, returns to scale are three type;

i- Increasing returns to scale
ii- Decreasing returns to scale
iii- Constant returns to scale

Increasing returns to scale:

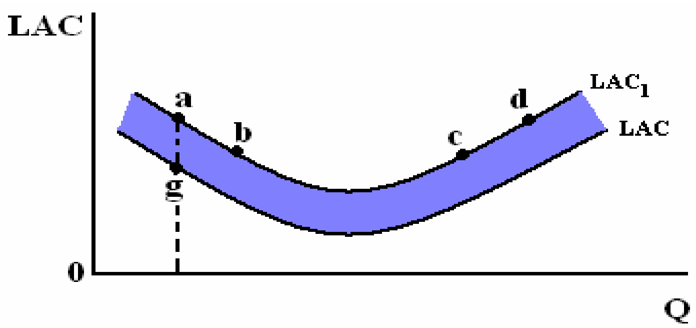
If we use 10 units of inputs and in return we get more production than increase in inputs is called *increasing returns to scale.* "The case when output grows proportionately more than inputs".

Decreasing returns to scale:

If by employing an additional unit of input, firm gets less production then it *decreasing returns to scale.*

Constant returns to scale:

When all inputs are increased in a given proportion and the output produced increases in the same proportion is called *constant returns to scale*.

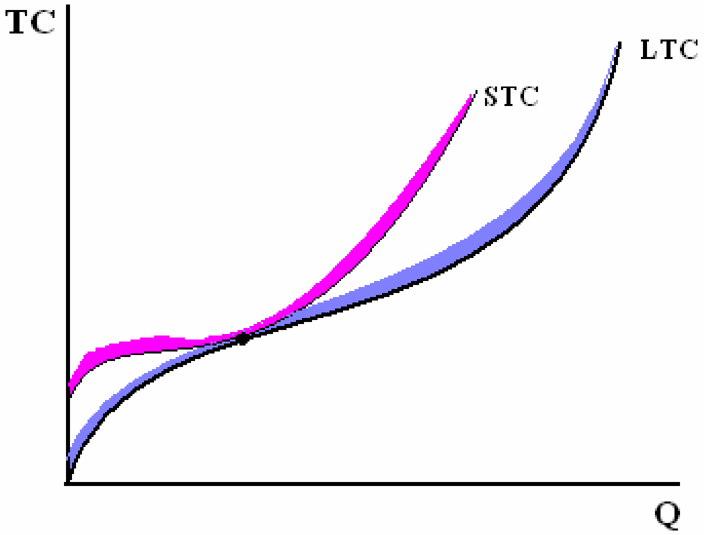


Relationship between short-run and long-run cost curves:

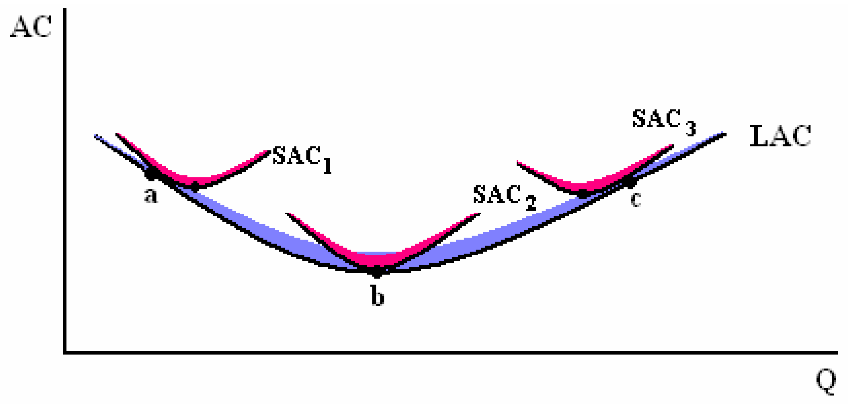
i- STC and LTC
ii- SAC and LAC
iii- SMC and LMC

STC and LTC:

LTC starts from origin because there is no fixed cost in long-run.

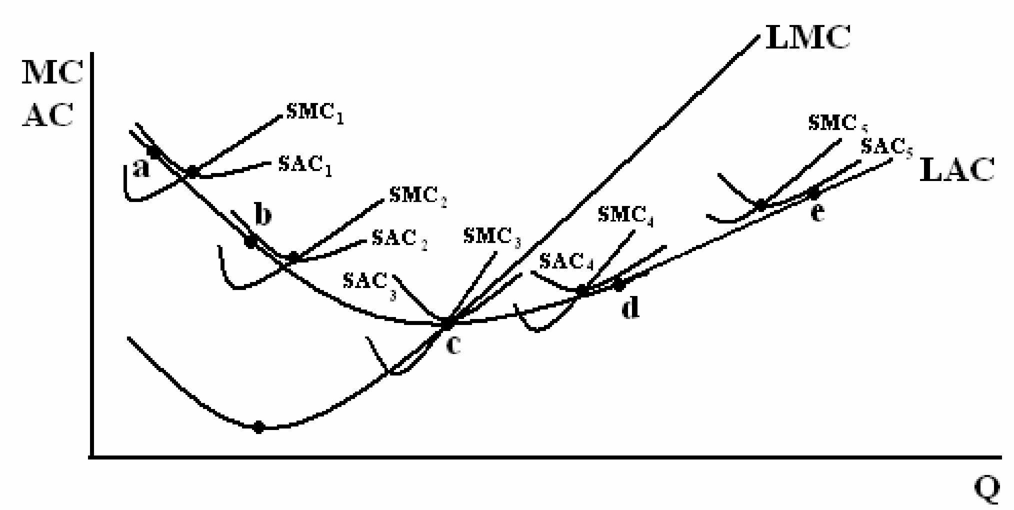


- When LTC increases at decreasing rate (before point "a"), STC will be tangent to it and at the tangency point behavior of both is same
- When LTC is at its inflection (i.e. point "a"), one STC will be tangent to it and at that tangency point STC has also its inflection point
- When LTC increasing at increasing rate (after "a" point), one STC is tangent to it and at that tangency point STC is also increasing at increasing rate



- At point "a": both LAC and SAC are decreasing
- At point "b": both LAC and SAC are minimum

- At point "c": both LAC and SAC are increasing



Production in Long-run

Production function:

$$Q = f(L, K)$$

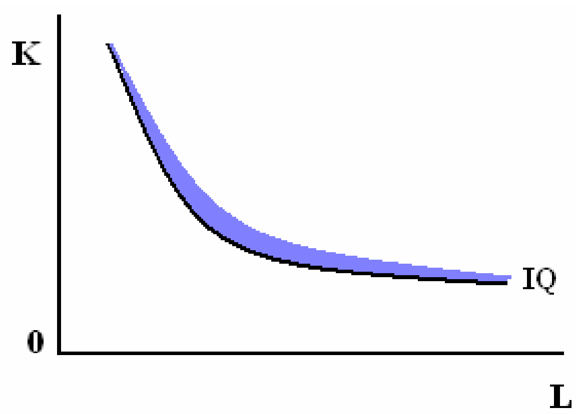
- Both capital and labor are variable in long-run

Concept of ISO-Quant curves:

(ISO product/Production indifference curves)

A curve along which same level of (production) output is shown with different combinations of labor and capital

"A showing different combination of labor and capital which gives us same level of output is called ISO-quant".



- It is the curve along which Q is constant.

$$\bar{Q} = f(L, K)$$

Marginal Rate of Technical Substitution (MRTS):

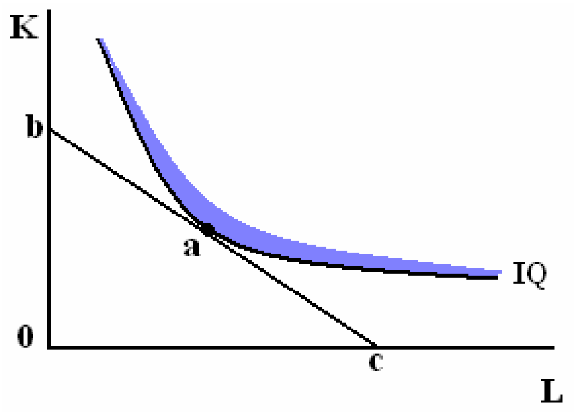
The marginal rate of technical substitution of L for K ($MRTS_{LK}$) is the amount of K that a firm can give up in order to increase the amount of L by one unit and still remains on the same ISO-quant.

$$MRTS_{LK} \text{ is also equal to } \frac{MP_L}{MP_K}$$

$$MRTS_{LK} = \frac{\Delta K}{\Delta L_{=1}}$$

How to find MRTS graphically?

Graphically MRTS is the slope of ISO-quant.



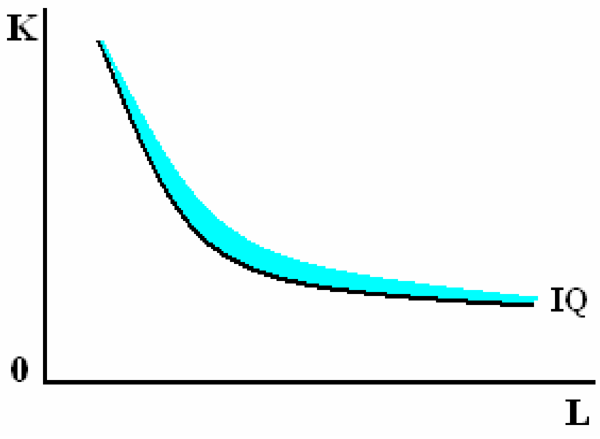
$$\text{slope of } ISO - \text{quant at point a} = \frac{Perpendicular}{Base} = \frac{0b}{0c}$$

Behavior of $MRTS_{LK}$ and slope of ISO-quant

- **i-** $MRTS_{LK}$ Diminishes
- **ii-** $MRTS_{LK}$ Constant
- **iii-** $MRTS_{LK}$ Increases

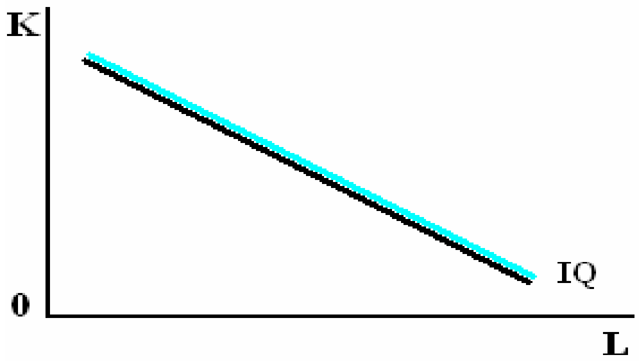
$MRTS_{LK}$ Diminishes:

For every additional unit of labor, smaller amount of capital is forgiving. In this case, ISO-quant will be convex to origin.



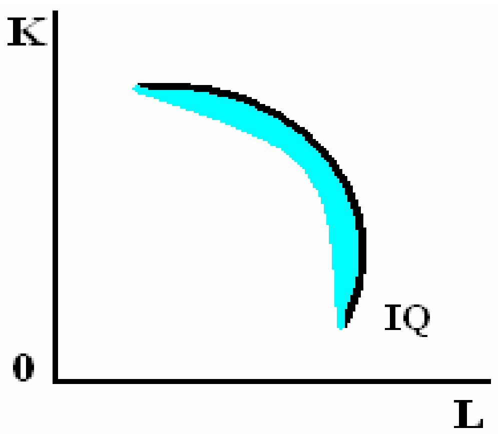
$MRTS_{LK}$ Constant:

For every additional unit of labor, fixed amount of capital is forgiving. In this case, ISO-quant will be a negatively slopped straight line.



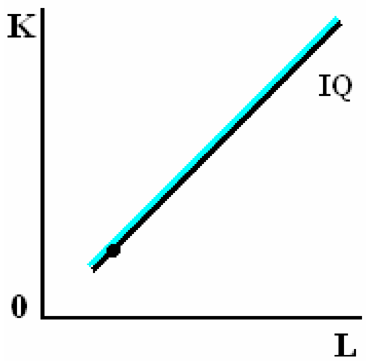
$MRTS_{LK}$ Increases:

For every additional unit of labor, larger amount of capital is forgiving. In this case, ISO-quant will be concave to origin.



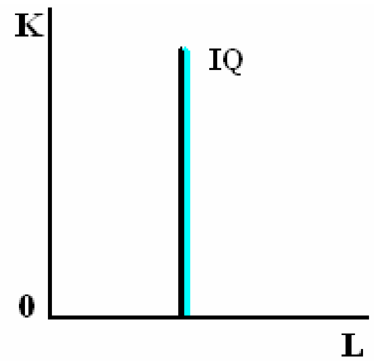
Few more shapes of ISO-quant

i- Positively slopped IQ



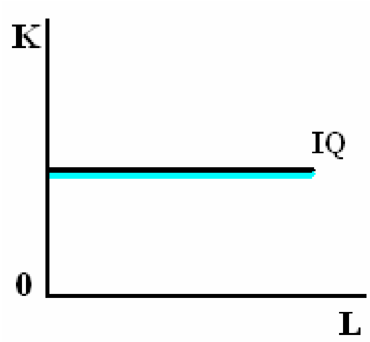
- Labor and capital are not perfect substitutes and in this case lower point is feasible.

ii- Vertical IQ



- It is not realistic in nature

iii- Horizontal IQ



- It is also not realistic in nature

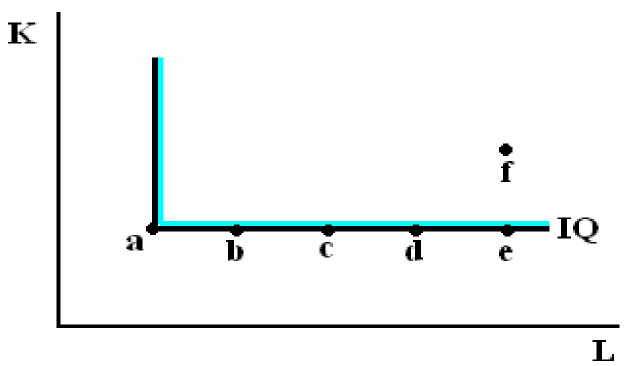
iv- Right-angled IQ

When both factors are used in a fixed proportion

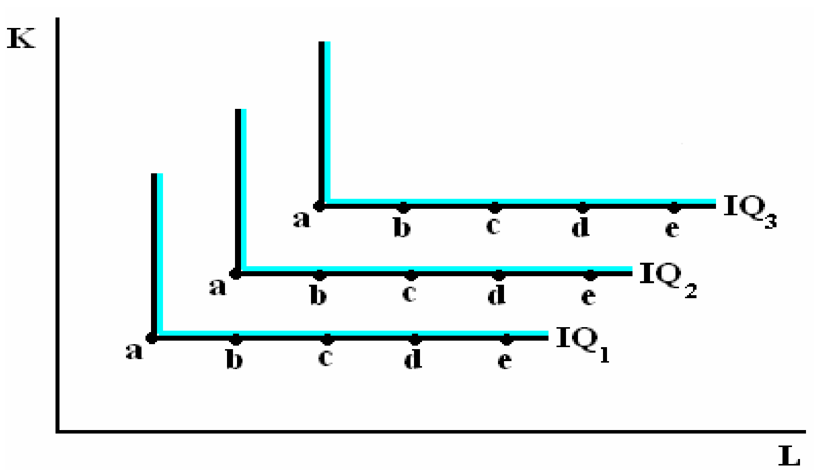
For example: we must use 5labor to operate 1 capital (machine).

- If a producer has 1 machine and 5 labors, then he will use all the amount of labor and capital.
- If the amount of labor is 6, 7, 8. 9, or 10 and machine is still 1, then these extra fire labor are not feasible for producer.
- If there are 10 labors and 1 machine is added to capital then producer will use all the amount of labor and capital.

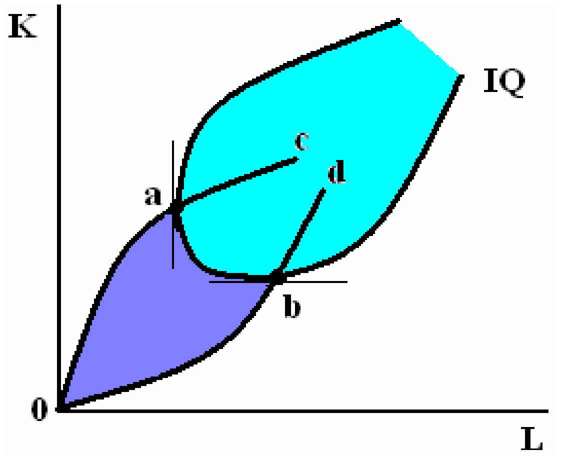
Only one feasible point



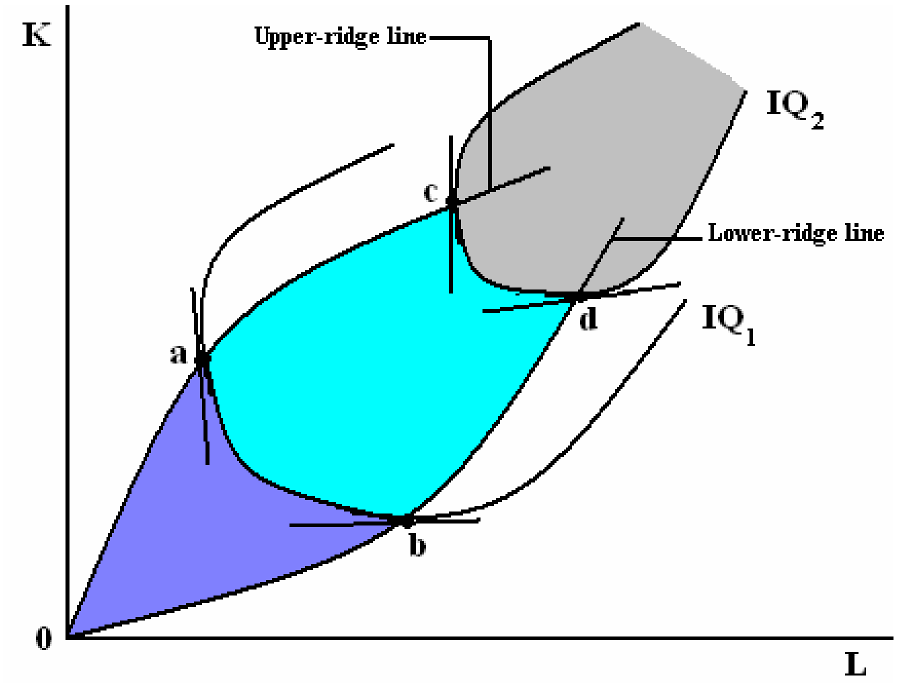
More than one IQ



v- A typical IQ



At point "a" slope of IQ = infinity
At point "b" slope of IQ = zero
0c is lower ridge line
0d is upper ridge line



- Feasible zone is in between of upper and lower ridge lines

Properties of IQ:

- **i-** Higher the curve greater the level of production
- **ii-** ISO-quant have negative slope if inputs are substitutes
- **iii-** IQ can touch the axes
- **iv-** IQ present everywhere at the plane

- **v-** IQ never intersects
- **vi-** In relevant range IQ are negative slopped

ISO-Cost line

A line showing various combinations of labor and capital among which the firm can hire any one, when wages (prices of labor), interest rate (prices of capital) and the cost of production are given.

The ISO-Cost equation is;

$$\bar{C} = \bar{w}.L + \bar{r}.K$$

Example:

C = 1000, w = 100, r = 100

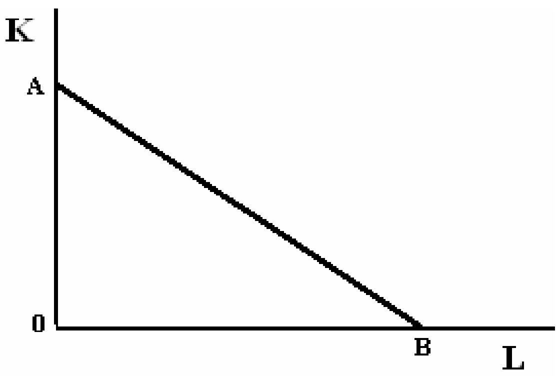
$$as \qquad \bar{C} = \bar{w}.L + \bar{r}.K$$

$$1000 = (100).L + (100).K$$

If L = 0 then K = 10

And

If K = 0 then L = 10



Slope of ISO-Cost line is P_L/P_K
Derivation of slope is same as the slope of budget line

AB is ISO-Cost line

Optimal Decision of production and input hiring

Case 1: Maximum output subject to given budget constraints

$$Max\ Q$$
$$S.T \quad C\ is\ given$$

Case2: Minimum cost subject to given output constraints

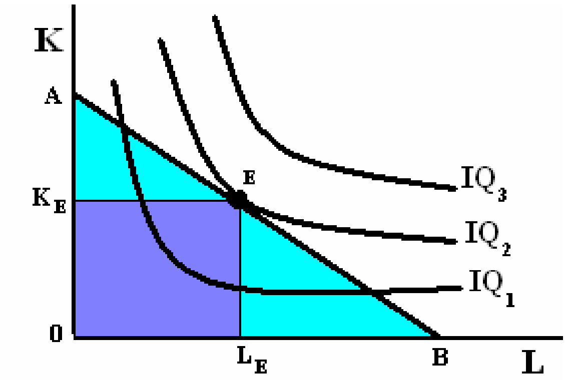
$$Min\ C$$
$$S.T \quad Q\ is\ given$$

With convex IQ

Case1:

Max Q
S.T C is given

ISO-Cost line is given

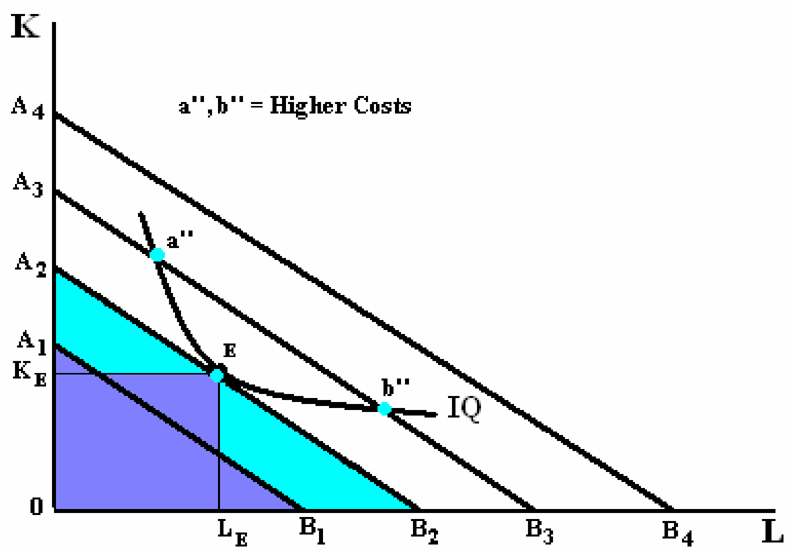


- Producer has a fix budget to produce maximum with his given cost

Case2:

Min C
S.T Q is given

ISO-Quant curve is given



- Output level is given to the producer. Producer will try to produce it at minimum cost

Equilibrium Conditions:

i- ISO-Cost line must tangent to IQ (necessary)
ii- Slope of ISO-Cost and slope of IQ must be same

Slope of IQ = slope of ISO-Cost

$$MRTS_{LK} = -\frac{P_L}{P_K}$$

iii- IQ must convex to origin (sufficient)

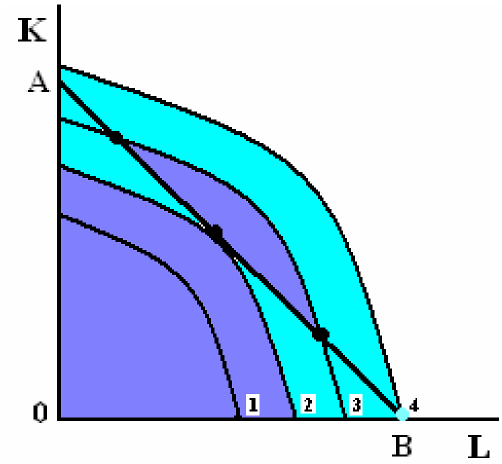
With concave IQ

Case1:

$Max\ Q$

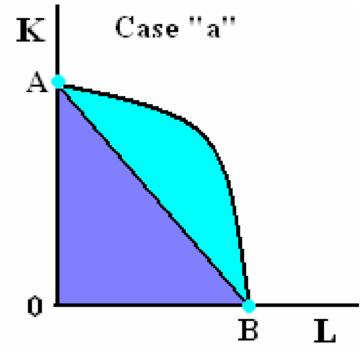
$S.T \quad C\ is\ given$

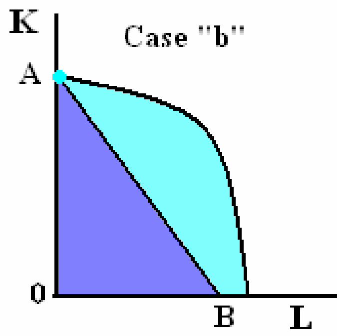
ISO-Cost line is given



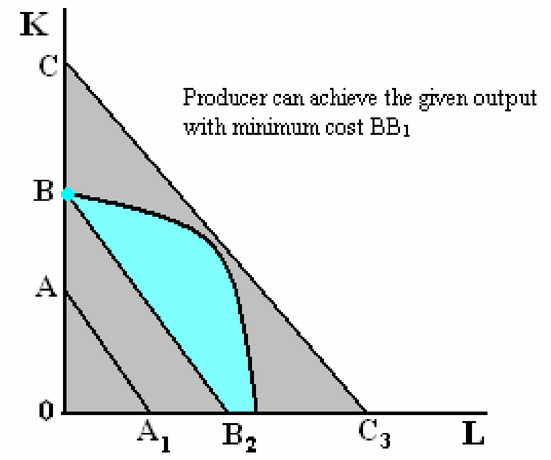
AB is the ISO-Cost line

It makes tangent to IQ_3 at point "c" production level can be fulfill at point "b" on the higher IQ. So producer will hire only labor for maximum level of production. So it is one input case and will have ***corner solution***.





- In case "a", producer will hire only capital or only labor for maximum level of production
- In "case "b", producer will hire only capital for maximum level of production



Case2:

Min C
S.T Q is given

ISO-Quant curve is given

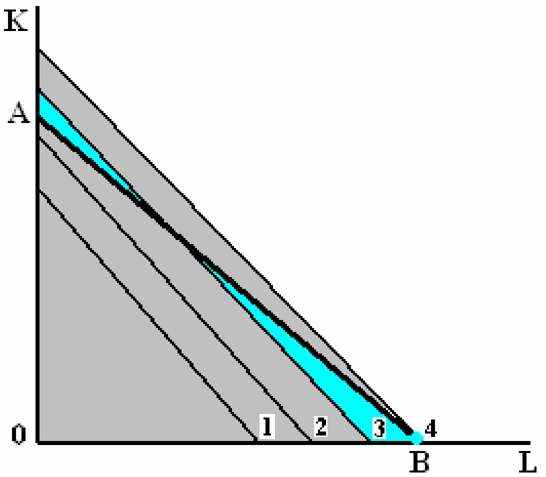
- Producer will hire only capital (K) for maximum level of production. IQ is given; producer can fulfill his level of production with given (BB_1) ISO-Cost line.

With Straight line IQ

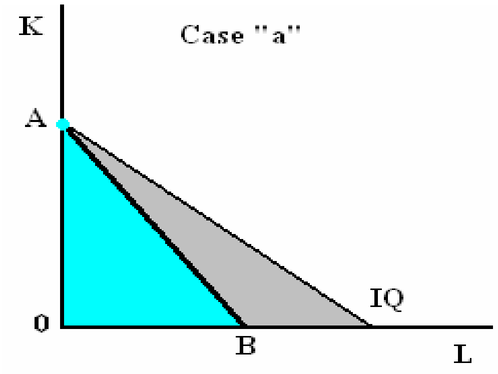
Case1:

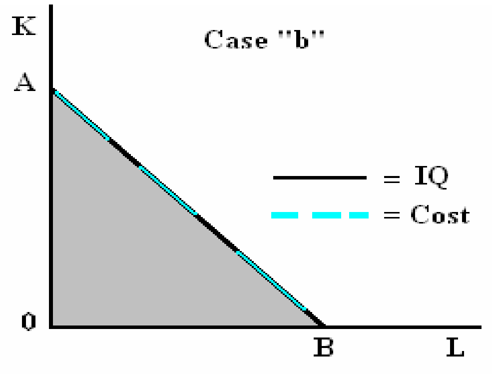
Max Q
S.T C is given

ISO-Cost line is given



- AB is ISO-Cost line and 1, 2, 3 and 4 are the IQ. Producer can take maximum output with hiring only labor at 0b, so it is also ***corner solution***.



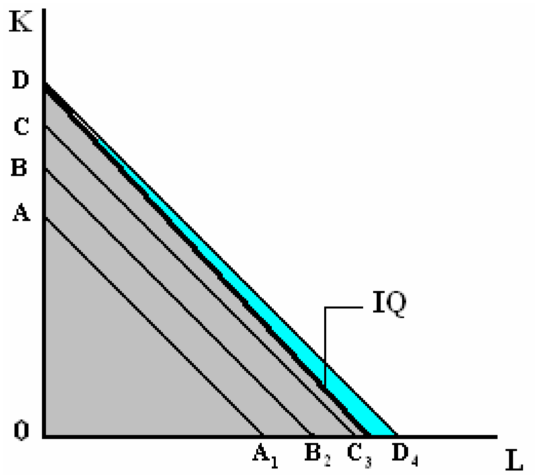


- In case “a”, producer will hire only capital (K) for maximum level of production
- In case “b”, producer has many equilibrium points by whish he can take maximum level of production

Case2:

$$\text{Min } C$$
$$S.T \quad Q \text{ is given}$$

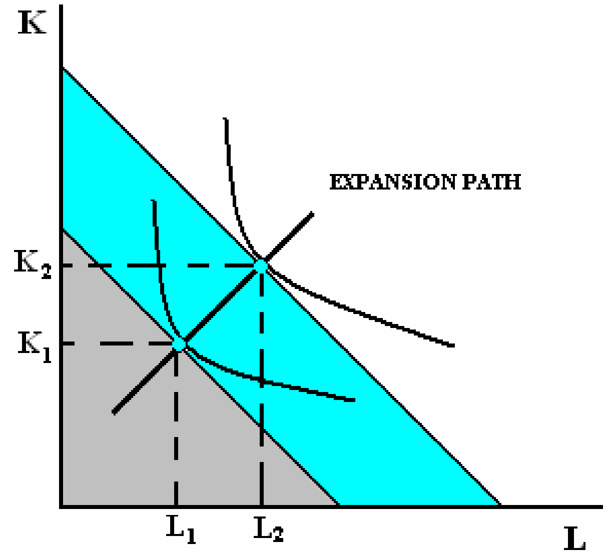
ISO-Quant curve is given



- Producer will hire only capital (K) for maximum level of production. IQ is given; producer can fulfill his level of production with given (DD_4) ISO-Cost line, so the given level of output can be achieved with minimum cost of DD_4.

Expansion Path

It is the path which shows the expansion of the firm, when input prices are given. If prices are given then ISO-Quant are parallel.



If the firm changes its total outlay (expenditure) while the prices of labor and capital remains constant then the firm's ISO-Cost line shifts;

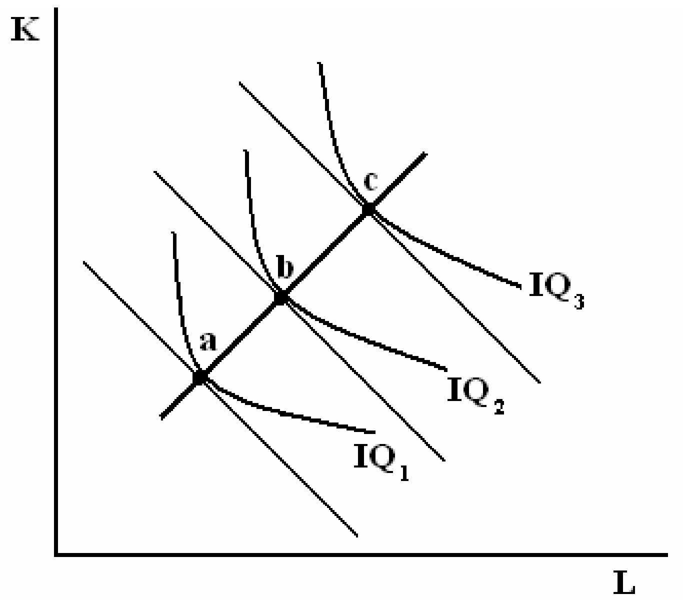
- Parallel to upwards if the expenditure is increased
- Parallel to downward if the expenditure decreased

The different ISO-Cost lines will be tangent to different IQ (which is equilibrium points). By joining these equilibrium points we get firm's ***expansion path*** (this idea is parallel to ICC).

ISOC lines

The locus of points on different IQ at which the marginal rate of technical substitution (MRTS) of factors of production is constant or slope of IQ's is constant.

The lines along which the slope of IQ is same



Properties of ISOC lines

i- Expansion path is a special case of ISOC lines
ii- Upper-ridge line lower-ridge line both are the special cases of ISOC lines
  - At upper-ridge line slope of IQ = zero
  - At lower-ridge line slope of IQ = infinity

iii- ISOC lines passes through equilibrium points

Market Conditions

Classification with respect to Time:

- Short-Period
- Long-Period

Classification with respect to Competition:

- Perfect Competition
- Imperfect Competition
  - Monopoly
  - Monopolistic Competition
  - Oligopoly
    - Non-collusive
    - Collusive

Perfect Competition:

Large number of buyers and sellers;

- The number of buyers and sellers are so large that grouping is not possible.
- Any single seller or buyer does not have any personal identity (no significance or no popularity).
- The number of buyers and sellers are so large that an individual buyer or seller can not influence on the market.
- No one can hold the big share of market.

Identical products;

- All the sellers have 100% same product.
- No product differentiation, no trade marks and no label.

Perfect knowledge;

- Every one of buyer and sellers has perfect knowledge of market conditions.

Unrestricted entry and exit;

Prices are same;

- All the buyers and sellers are price takers (not price setters or price acceptors).

Monopoly:

Opposite of perfect competition

- Single seller.
- No close substitute of his product (i.e. PTV, PIA, PTCL etc).
- Restricted entry or barriers to entry.
- Firm is price setter.

Monopolistic Competition:

Idea between perfect and monopoly competition

- Enough number of sellers.
- Product differentiation (presentation, label and quality can be different but te product is same).
- Advertisement cost (selling cost).
- Small price variation is possible because products are different
- Unrestricted entry.

Oligopoly:

In oligopoly competition, there are few numbers of sellers, so few if they can make groups then they called *collusive* oligopoly. If they do not make any grouping then they called as *non-collusive* oligopoly.

Revenue Structure of the firm

Revenue:

It is the amount which a firm gets after selling its products in the market.

Forms of Revenue:

i- Average Revenue

$$AR = \frac{TR}{Q}$$

$$AR = \frac{P.Q}{Q}$$

$$\therefore \quad AR = P$$

ii- Marginal Revenue

$$\therefore \quad MR = \frac{\Delta TR}{\Delta Q_{=1}}$$

iii- Total Revenue

$$\therefore \quad TR = P.Q$$

Behavior of TR, MR and AR in different market structures:

✓ **In Perfect Competition**

As we know that in perfect competition, price is constant, so

$$\therefore \quad TR = \bar{P}.Q$$

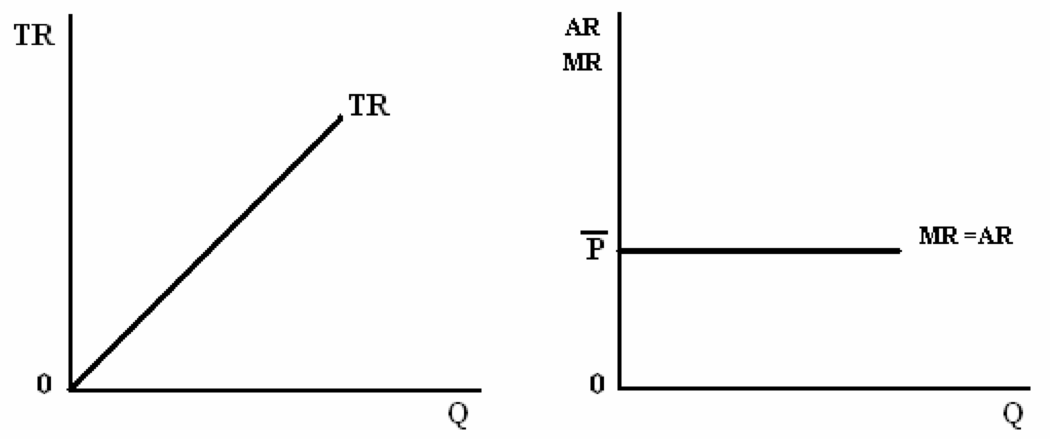
$$\therefore \quad AR = \bar{P}$$

$$\therefore \quad MR = \bar{P}$$

Example:

P	Q	TR	MR	AR
5	0	0	--	5
5	1	5	5	5
5	2	10	5	5
5	3	15	5	5

$$Hence \quad MR = AR = P = constant$$

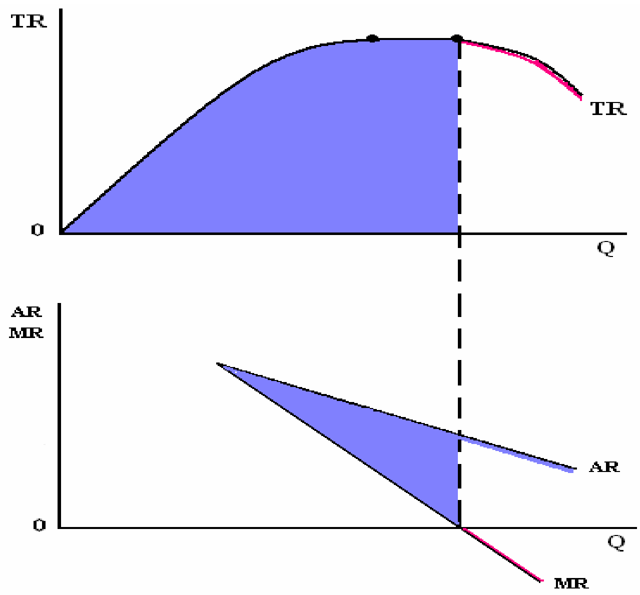


✓ **<u>In Monopoly Competition</u>**

Price is not constant in monopoly. Firm can increase its quantity sold by reducing prices.

Example:

P	*Q*	*TR*	*MR*	*AR*
6	0	0	--	--
5	1	5	5	5
4	2	8	3	4
3	3	9	1	3
2	4	8	-1	2
1	5	5	-3	1



Forms of Profit:

i- **Normal Profit**

Minimum amount which a firm needs for survival

$$\therefore \quad TR = TC$$

ii- **Sub-Normal Profit**

It is the revenue which is less than cost of firm

$$\therefore \quad TR < TC$$

iii- **Super-Normal profit**

It is the revenue which is more than cost of firm

$$\therefore \quad TR > TC$$

Explicit Cost:

- The cost of those resources of the firm which are purchased from the market

Implicit Cost:

- The cost on those resources which are owned by the firm, normal profit only covers the implicit and explicit costs of the firm

Equilibrium of the Firm

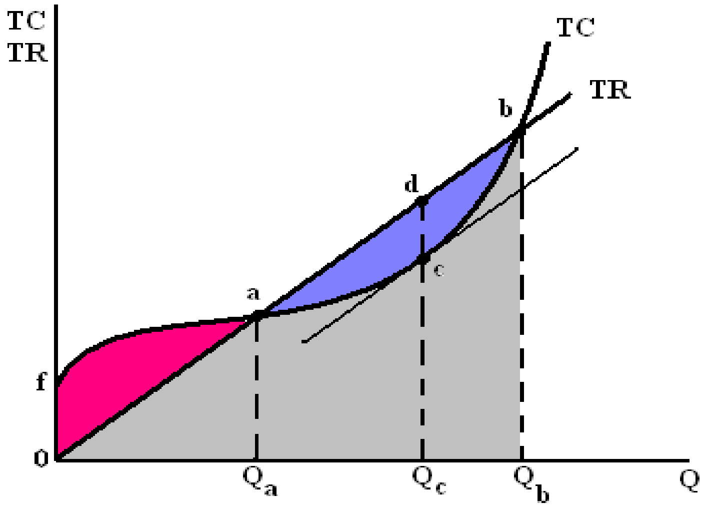
A firm is said to be in the state of equilibrium if under the given conditions, profit of the firm should be maximum of its cost.

1- Under Perfect competition

(Short-run Analysis)

- As we know that in the perfect competition all firms are price takers, and in short-run some cost is fixed and some is variable.
- In long-run there is unrestricted entry or exit for every firm but in short-run fix cost forces that are fixed not to exit (if loss is more than fix cost then firm will exit).
- Entry of new firm is banned in short-run

Bench Mark: In short-run, entry of new firm is not possible. Exit of any firm is also possible due to fixed cost.



- Before Q_a point: TR < TC it means loss
- At Q_a point: TR = TC it means normal profit
- After Q_a point and before Q_b point: TR > TC it means Super-normal profit
- At Q_b point: TR = TC it means normal profit
- After Q_b point: TR < TC it means loss
- "a" and "b" points are bench mark points (TR = TC)
- Equilibrium points of a firm lies between point "a" and 'b"

Equilibrium condition

$$\pi = TR - TC$$

Differentiate with respect to Q

$$\frac{d\pi}{dQ} = \frac{dTR}{dQ} - \frac{dTC}{dQ}$$

For maximization

$$as \qquad \frac{d\pi}{dQ} = 0$$

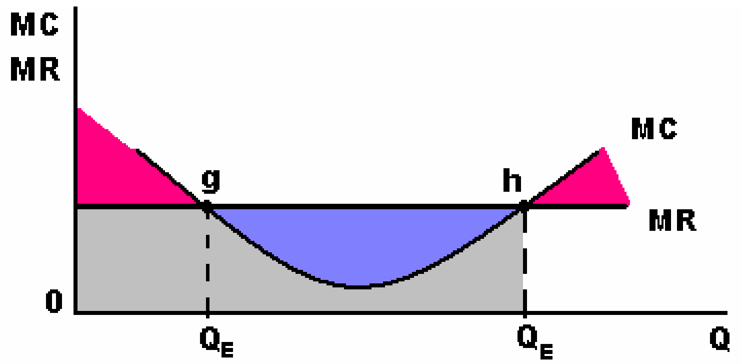
$$\therefore \qquad 0 = \frac{dTR}{dQ} - \frac{dTC}{dQ}$$

$$\therefore \qquad \frac{dTR}{dQ} = \frac{dTC}{dQ}$$

MC = MR (condition for maximization, this is necessary condition)

Slope of TC = slope of TR

So we will plot a tangent to TC which will be parallel to TR (slope of TC and TR are same)



- MR = MC is not a sufficient condition for equilibrium condition, there are two points "g" and "h" where MC = MR. sufficient condition for equilibrium is:
- Slope of MC > slope of MR
- So these two conditions of firm's equilibrium are for all types of market.

For Perfect competition

i- MC =MR
- As in perfect competition
- MR =AR =P
- So the equilibrium condition is MC = MR =AR = P

ii- Slope of MC > slope of MR
- As in perfect competition
- Slope of MR = 0
- So equilibrium condition is slope of MC > 0

In short-run there are five cases of equilibrium under perfect competition.

i- P > ATC (Super-normal profit)

ii- P = ATC (Normal profit)

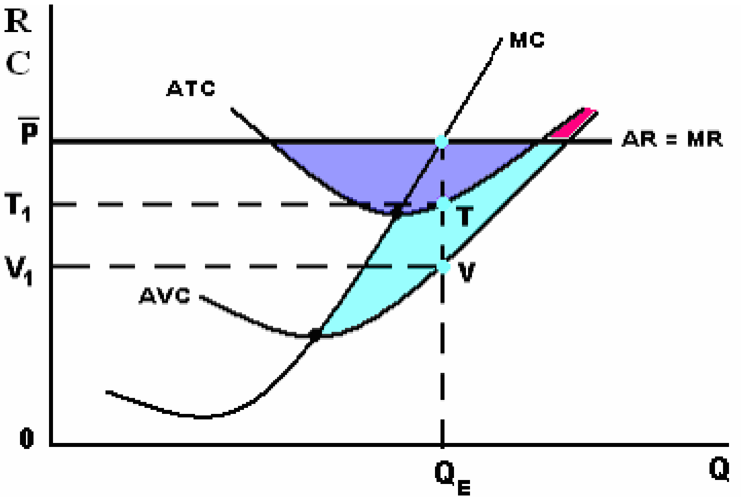
iii- AVC < P < ATC (normal loss, if loss is less than FC)

iv- P = AVC (loss of fixed cost)

v- P < AVC (loss is greater than FC, shut-down point)

✓ **Super-normal profit:**

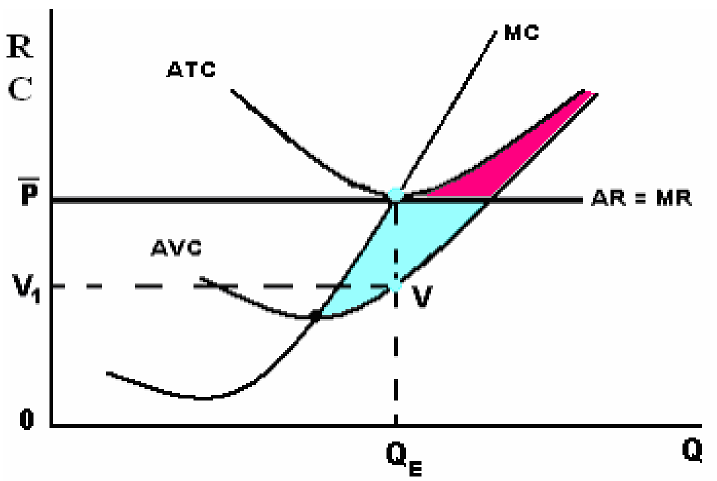
P > ATC



$$TR = 0\bar{P}EQ_E$$
$$TC = 0T_1TQ_E$$
$$\pi = TR - TC$$
$$= 0\bar{P}EQ_E - 0T_1TQ_E$$
$$\therefore \quad \pi = \bar{P}ET_1T$$

✓ **Normal profit:**

P = ATC

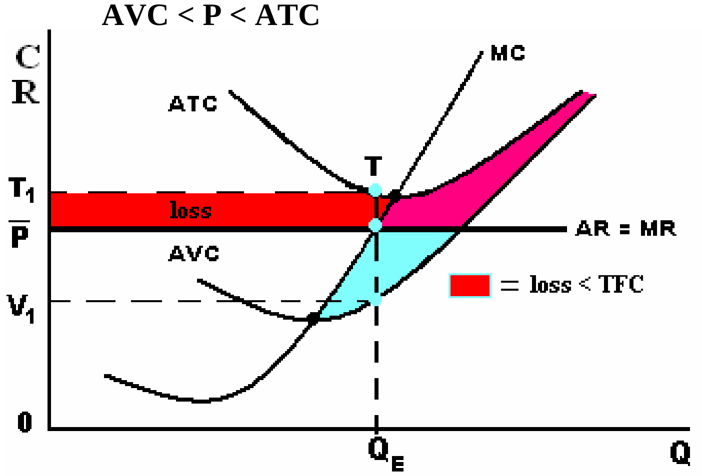


$$TR = 0\bar{P}EQ_E$$
$$TC = 0\bar{P}EQ_E$$
$$\pi = TR - TC$$
$$\pi = 0$$

$$as \quad TR - TC = 0$$
$$so \quad TR = TC$$

✓ **Normal Loss:**

AVC < P < ATC

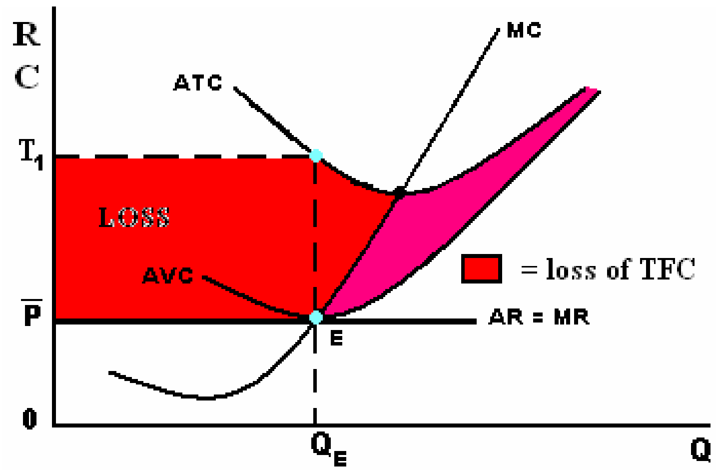


$$TR = 0\bar{P}EQ_E$$
$$TC = 0T_1TQ_E$$
$$\pi = TR - TC$$
$$= 0\bar{P}EQ_E - 0T_1TQ_E$$
$$\therefore \quad \pi = -\bar{P}ET_1T$$
$$\therefore \quad \pi = \text{TFC}$$

- In this case, there will be loss of TFC (less than TFC)

✓ **Loss off Fixed cost:**

P = AVC

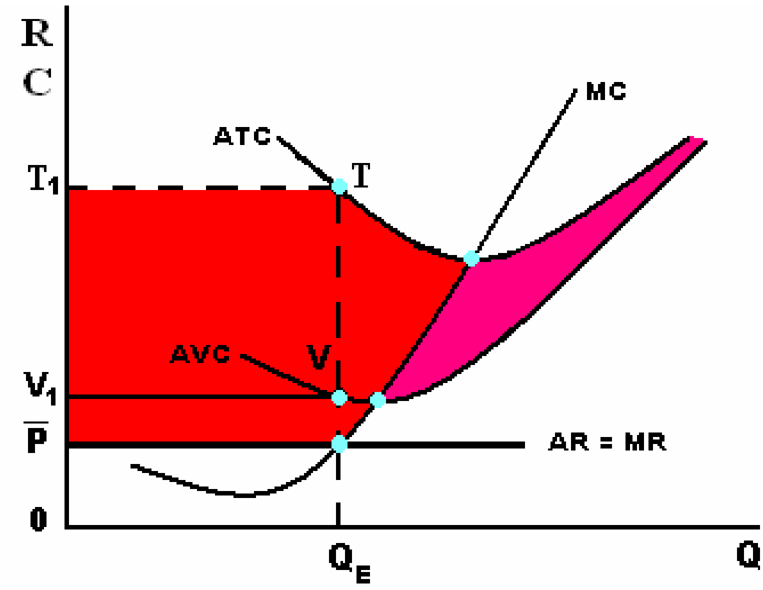


$$TR = 0\bar{P}EQ_E$$
$$TC = 0T_1TQ_E$$
$$\pi = TR - TC$$
$$= 0\bar{P}EQ_E - 0T_1TQ_E$$
$$\therefore \quad \pi = -\bar{P}ET_1T$$
$$\therefore \quad \pi = \text{TFC}$$

- In this case loss equal to TFC.

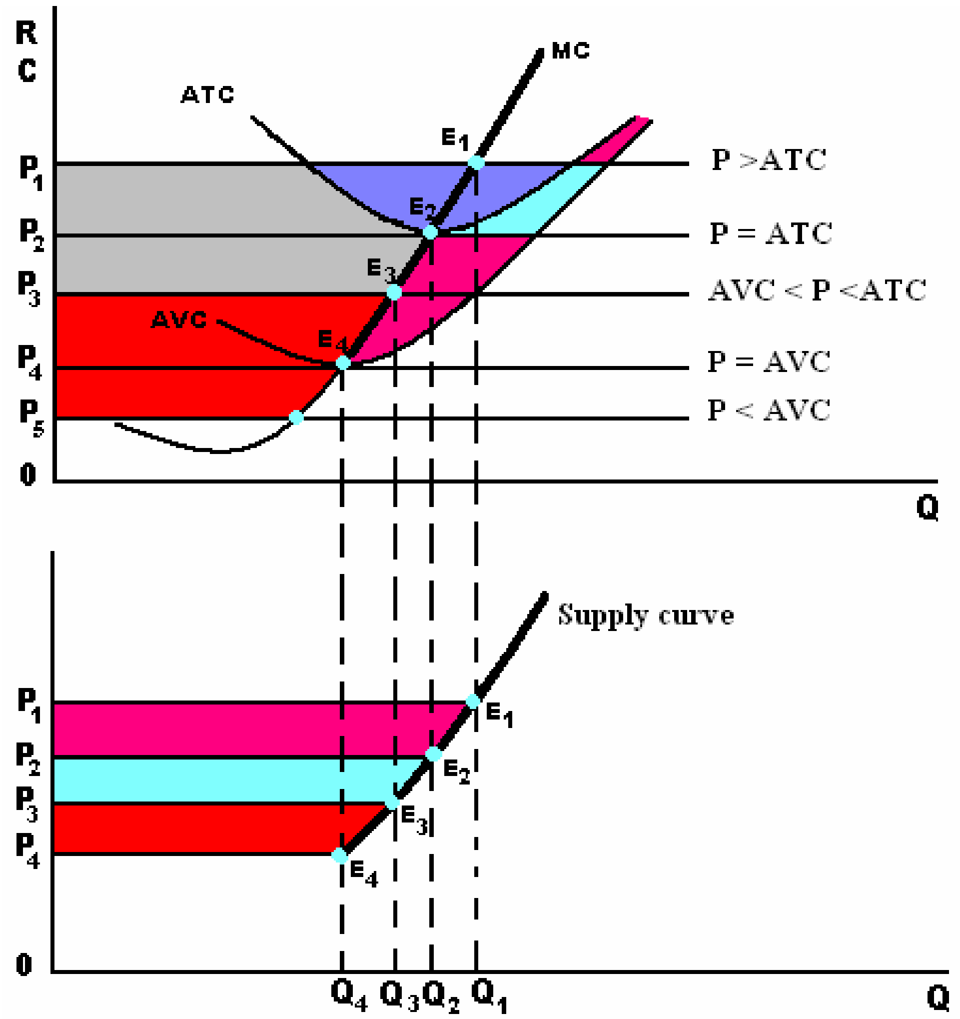
✓ **Shut-down point:**

P < AVC



$$TR = 0\bar{P}EQ_E$$
$$TC = 0T_1TQ_E$$
$$\pi = TR - TC$$
$$= 0\bar{P}EQ_E - 0T_1TQ_E$$
$$loss = -\bar{P}ET_1T \quad (loss > TFC)$$

Supply Curve of the firm under Perfect Competition
(In short-run analysis)



- Firm will not operate below E_4
- Supply curve is the part of MC which is above or equal to AVC

2- Under Perfect competition

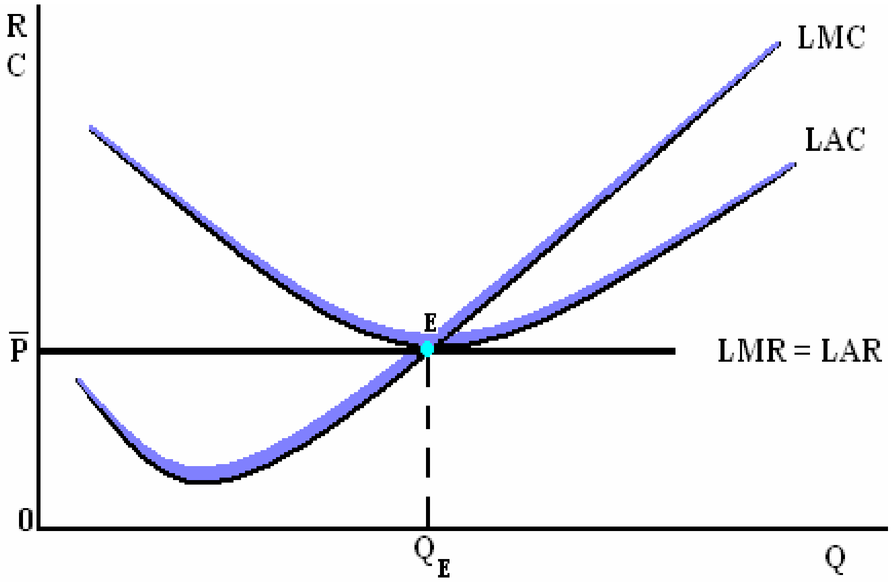
(In Long-run analysis)

- There is no fixed cost in long-run analysis because both the factors of production capital and labor are variables in long-run, so in long-run, exit of a firm is also possible because period is long enough.

a. If P > LAC, firm is earning super-normal profit, which is always attracts the new firms to enter in the market; Supply increases → S > D → excess of goods → price of good decreases → until it gets equal to LAC. Once P = LAC, then entry of firms will stop.

b. If P < LAC → loss to firm → firms will exit (which are financially weak) → supply decreases → S < D → shortage of goods → price of good increases → until it gets equal to LAC. Once P = LAC, then exit of firms will stop.

- Hence, in long-run, a firm ultimately earns normal profit
- Firm will produce at the minimum of LAC



Supply Curve of a firm under Perfect Competition

(In Long-run analysis)

In long-run, there can be three types of supply curves

i- Increasing Cost Industry

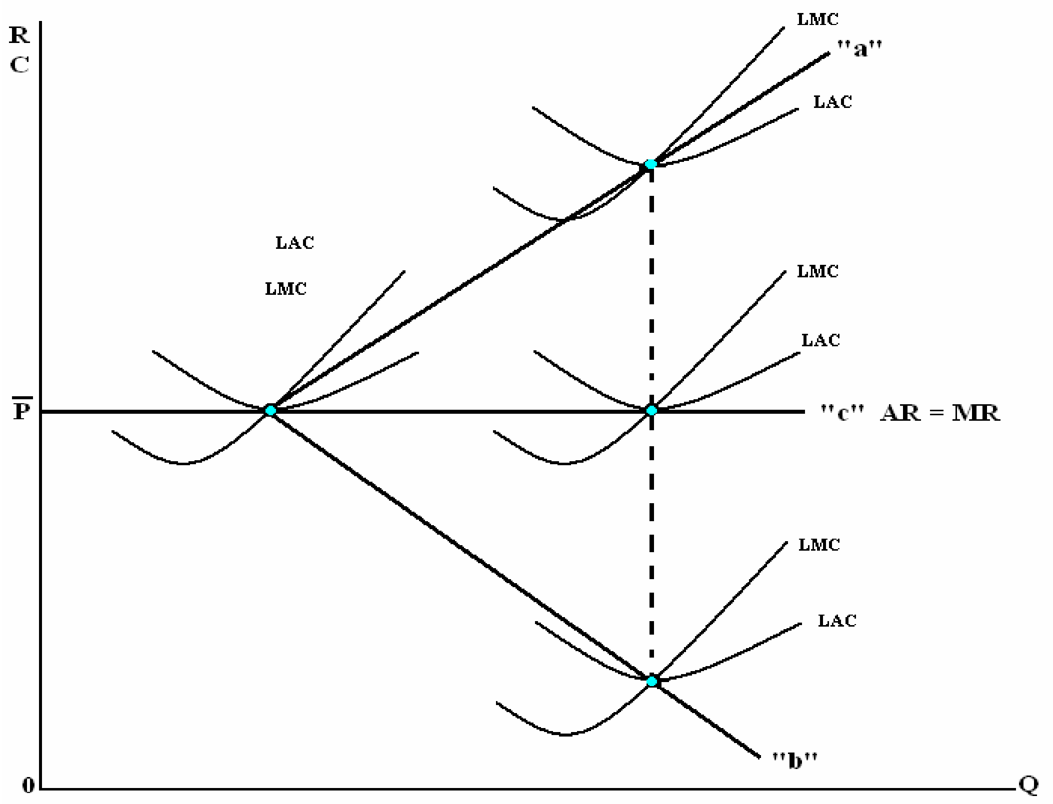
If any industry expands in such a way its per-unit cost increases

ii- Decreasing Cost Industry

If any industry expands in such a way its per-unit cost decreases

iii- Constant Cost Industry

If any industry expands in such a way its per-unit cost remains same or constant



- "a" is the supply curve of "*increasing cost industry*"
- "b" is the supply curve of "*decreasing cost industry*"
- "c" is the supply curve of "*constant cost industry*"

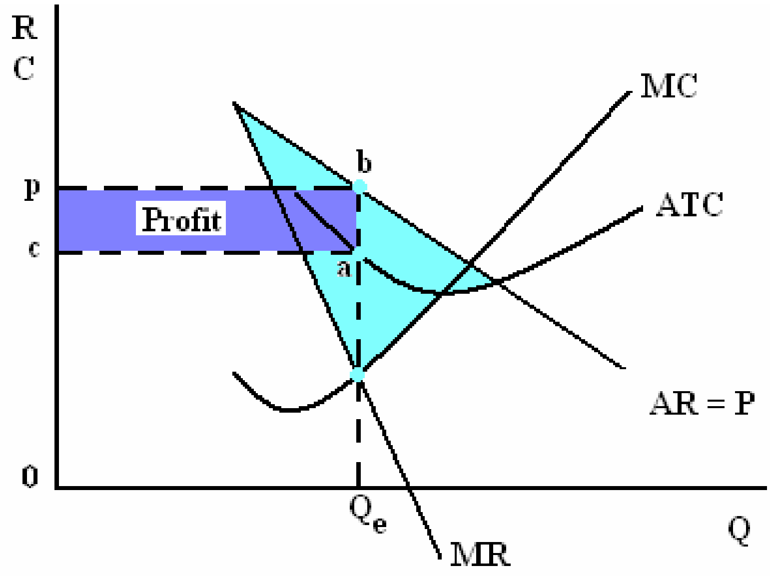
Monopoly

(In short-run analysis)

- In short-run, there are four case of equilibrium of firm under the monopoly
  - **i-** P > ATC (Super-normal profit)
  - **ii-** P = ATC (Normal profit)
  - **iii-** AVC < P < ATC (normal loss, if loss is less than FC)
  - **iv-** P = AVC (loss of fixed cost)

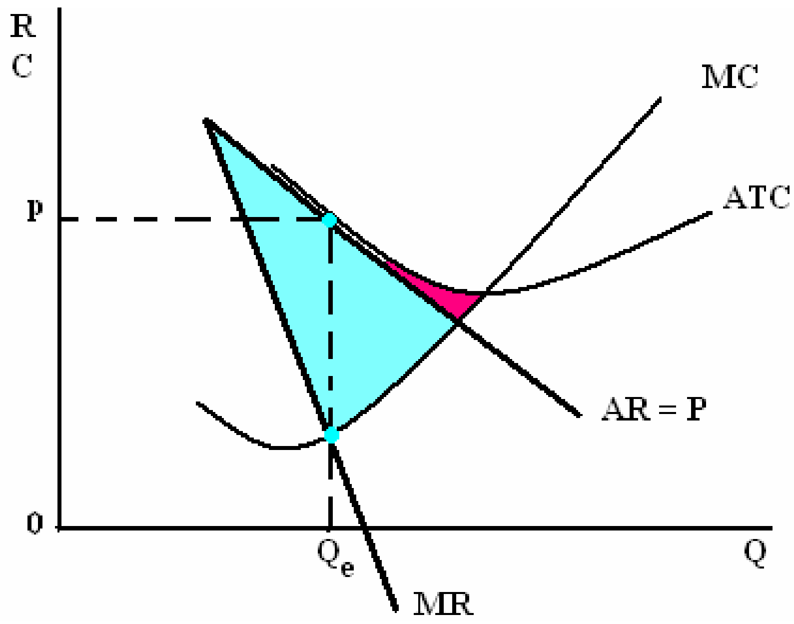
✓ **Super-normal profit:**

P > ATC



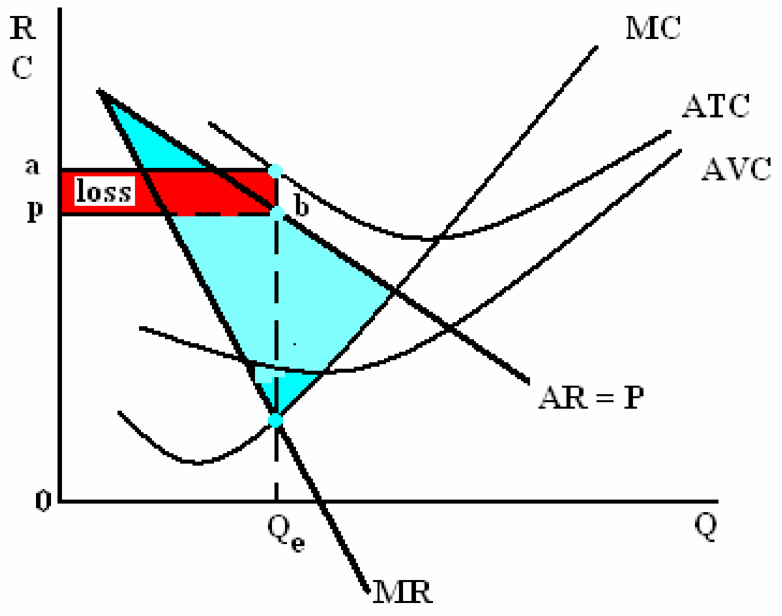
✓ **Normal profit:**

P = ATC



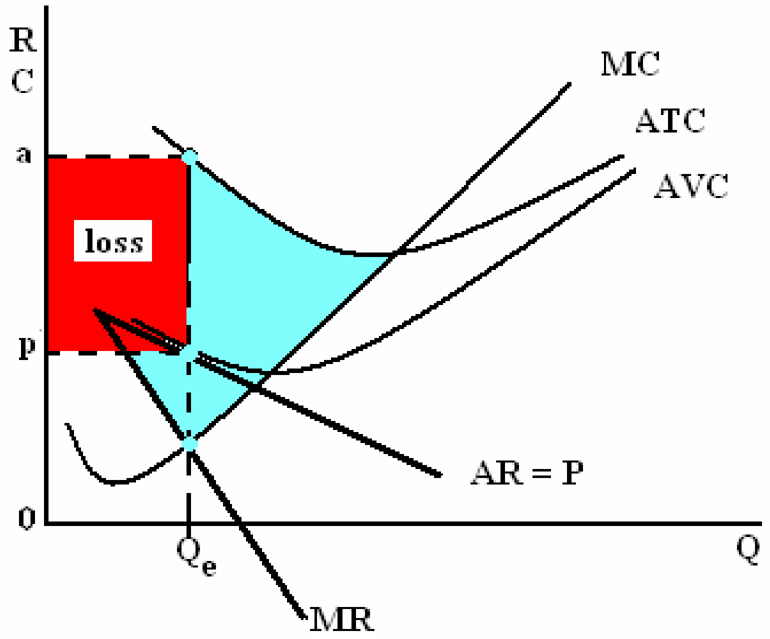
✓ **Normal loss:**

AVC < P < ATC



✓ **Loss of TFC:**

P = AVC

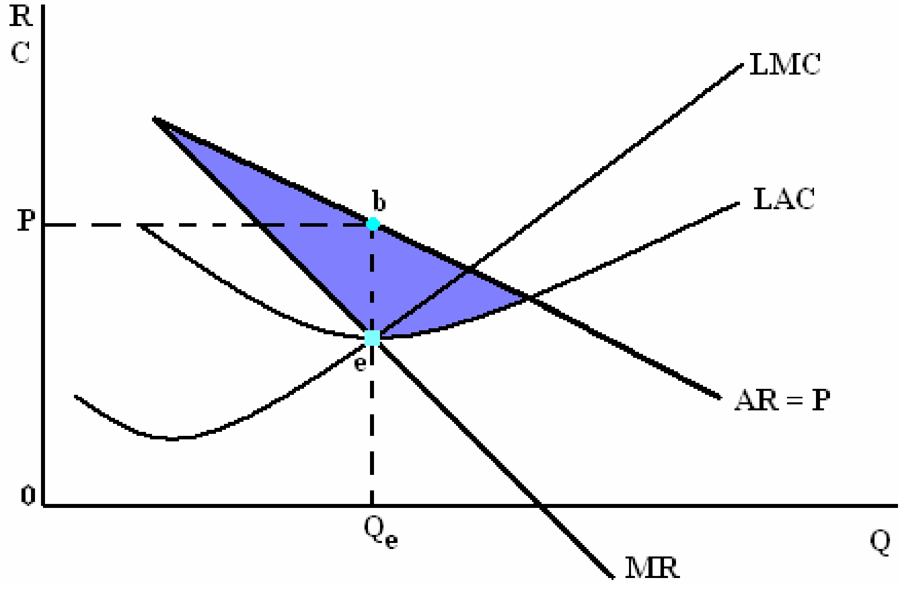


Monopoly:

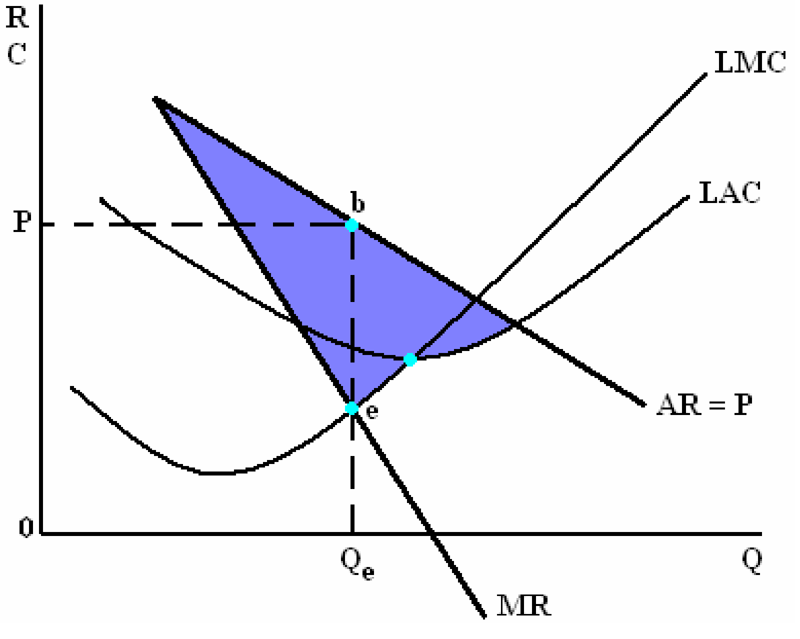
(In long-run analysis)

In long-run, a monopolist always earns super-normal profit (P > LAC)

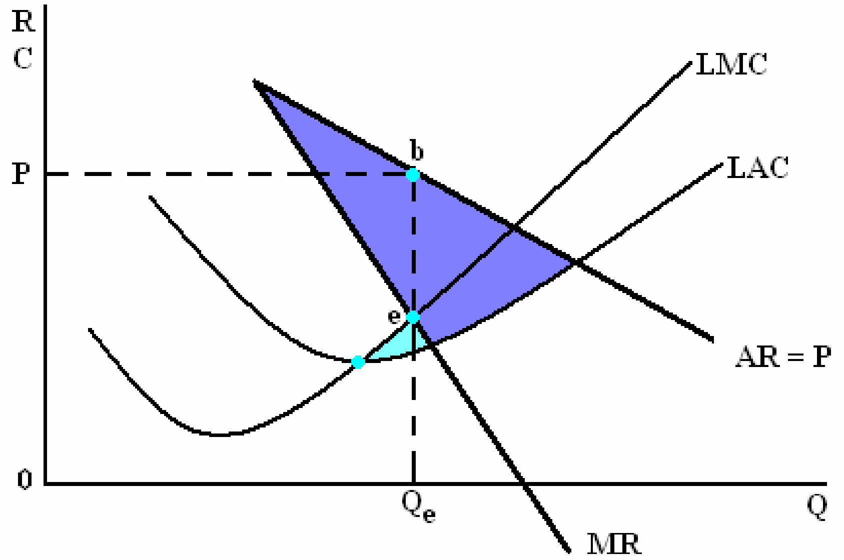
i- Optimum (Equilibrium at minimum of LAC)



ii- Sub-optimum (Equilibrium at falling portion of LAC)



iii- Super-optimum (Equilibrium at increasing portion of LAC)



Price Discrimination (for a monopolist)

Concept:

Selling the same product in different prices at same time is called *Price Discrimination*.

Forms/Shapes of Price Discrimination:

Price discrimination can be formed inn different shapes;

- Price discrimination with respect to ***Person***
- Price discrimination with respect to ***Age***
- Price discrimination with respect to ***Place***
- Price discrimination with respect to ***Use***
- Price discrimination with respect to ***Quantity purchased***

Degrees of Price Discrimination:

There are three degrees of price discrimination

1- Price discrimination of 1^{st} degree

It leaves no consumer surplus

Example:

If Price = 6 the Q_d = 4

Expenditure = 6*4 = 24

1^{st} unit sale: 1*9 = 9

2^{nd} unit sale: 1*8 = 8

3rd unit sale: 1*7 = 7

4th unit sale: 1*6 = 6

Total = 30 .

- When unique price is charged then expenditure = 24
- When different prices charged the expenditure = 30

So, consumer surplus = 30 – 24 = 6

- But price discrimination of 1st degree leaves no consumer surplus. It means producer charges different prices rather than unique price.

2- Price discrimination of 2nd degree

Pricing by parts

Example:

For 1st 10 units: P = Rs. 20

For 10 – 20 units: P = Rs. 18

For more than 20 units: P = 15

Producer charges different prices for different ranges

Example:

WAPDA charges different prices for different ranges (units of electricity consumed).

Units of Electricity	Per-unit charges
0 – 100	Rs. 3/-
101 – 150	Rs. 4/-
151 – 250	Rs. 5/-
251 – 500	Rs. 6/-
Above 500	Rs. 7/-

If consumer used 300 units, then how much producer charged?

For 1st 100 units: 3*100 = Rs. 300

For next 50 units: 4* 50 = Rs. 200

For next 100 units: 5*100 = Rs. 500

For next 50 units: 6*50 = Rs. 300

Then total expenditure = Rs. 1300

3- **Price discrimination of 3rd degree**

It is based on market segmentation; normally it is geographically but not necessary

Restriction:

- There should be solid market segmentation, i.e. price discrimination becomes possible only when the different parts of the monopolist's markets are separated from each other.
- Consumer of one market should not be able to purchase the commodity in low price market and resale it in high price market.

Conditions:

a- **Multiple price elasticities of demand**

- Necessary condition/ First-order condition
- 1st Sector: P decreases → Q_d increases → revenue increases
- 2nd Sector: P decreases → Q_d decreases → revenue decreases

If two markets have same elasticity of demand then policies are not useful

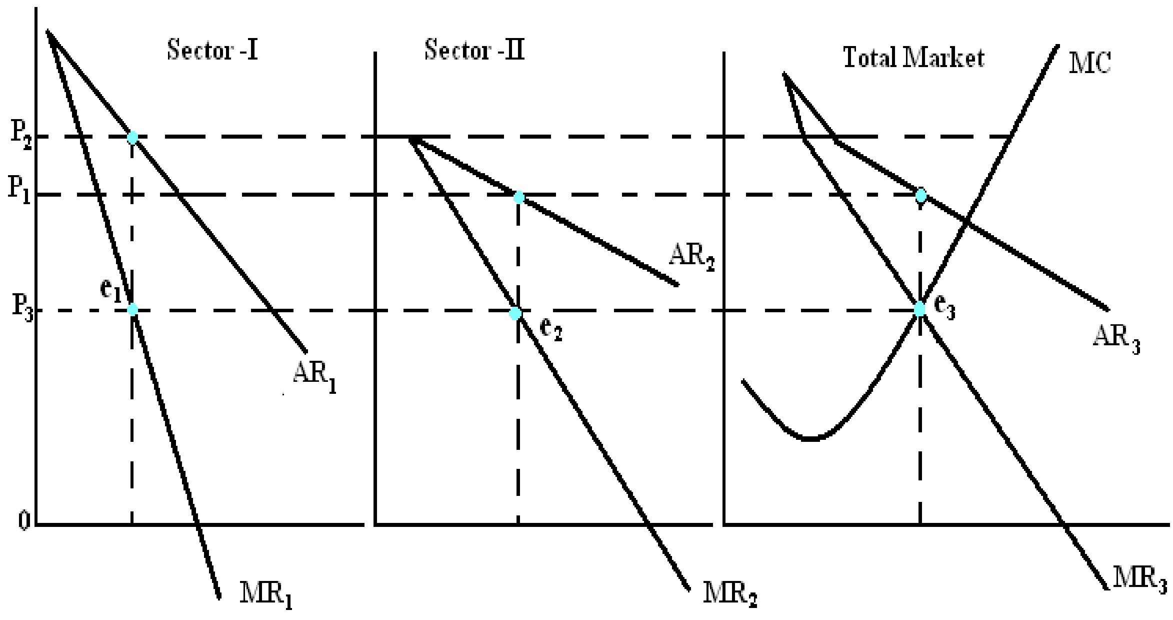
Example:

Situation – 1		Situation – 2	
Lahore	**Karachi**	**Lahore**	**Karachi**
E_p = - 1	E_p = - 1	E_p = - 1	E_p = - 1
Qd = 1000	Qd = 100	Qd = 500	Qd = 200
P = Rs. 2/-	P = Rs. 2/-	P = Rs. 4/-	P = Rs. 1/-
Revenue = Rs. 20000	Revenue = Rs. 200	Revenue = Rs. 20000	Revenue = Rs. 200
TR = 20000 + 2000 = 22000		**TR = 20000 + 2000 = 22000**	

When elasticities are same there will be no response

b- **Higher price should be charged in market with low price elasticity and low price should be charged in the market with high price elasticity.**

- Sufficient condition/ second-order-condition
- If condition (**b**) does not hold the price discrimination is not useful because response should be different and beneficial.



Monopolistic Competition

Concept of Proportional and perceived demand curve

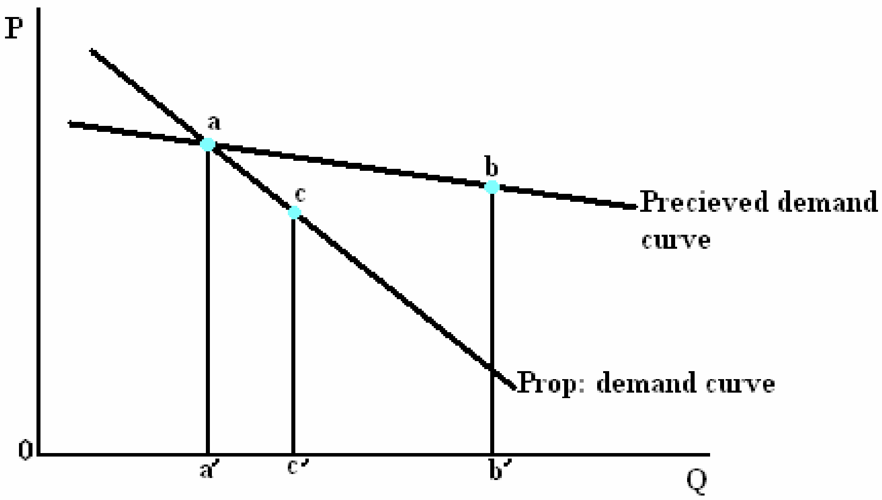
At any specific price, the proportional demand curve gives the representative firm's share of the total market quantity demanded, on the assumption that all firm's charge same price

Example:

Let the price of a firm's product is Rs. 15/- and Q_d of this product is 20 units. Firm wants to increase the demand, so it decreases the price of that product to Rs. 12/- (with thinking that demand will increase to 40 units). On the basis of firm's thinking the demand curve is formed. It is called "***Perceived demand curve***".

But in reality, due to response of other firms the demand does not increase as the firm's thinking. It only increases to 25 units. This real demand curve is called "***proportional demand curve***".

We explain this example with the help of following diagram;



- From 0 to b' = firm's desired demand
- From 0 to c' = in reality, firm's demand changes due to response of other firms

Equilibrium condition:

- 1st condition: MR = MC
- 2nd condition: slope of MC > slope of MR
- 3rd condition: Proportional demand curve and perceived demand curve should interest each other.

Equilibrium in short-run:

In short-run, there are four cases of equilibrium of a firm under the monopolistic competition.

i- P > ATC (super-normal profit)

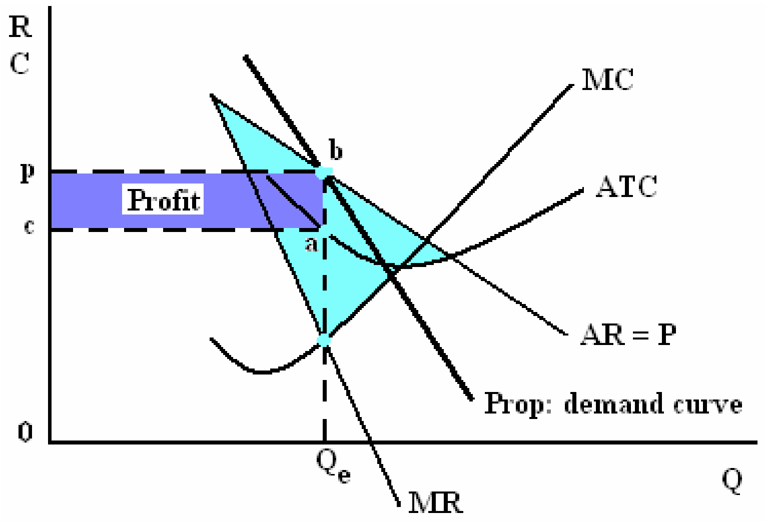
ii- P = ATC (normal profit)

iii- AVC < P < ATC (loss or partial loss of TFC)

iv- P = AVC (loss of TFC)

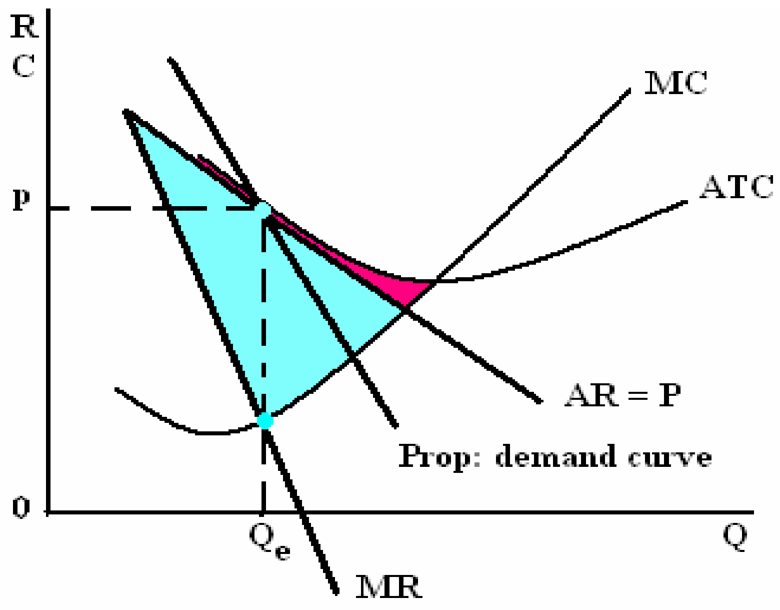
✓ **Super-normal profit:**

P > ATC



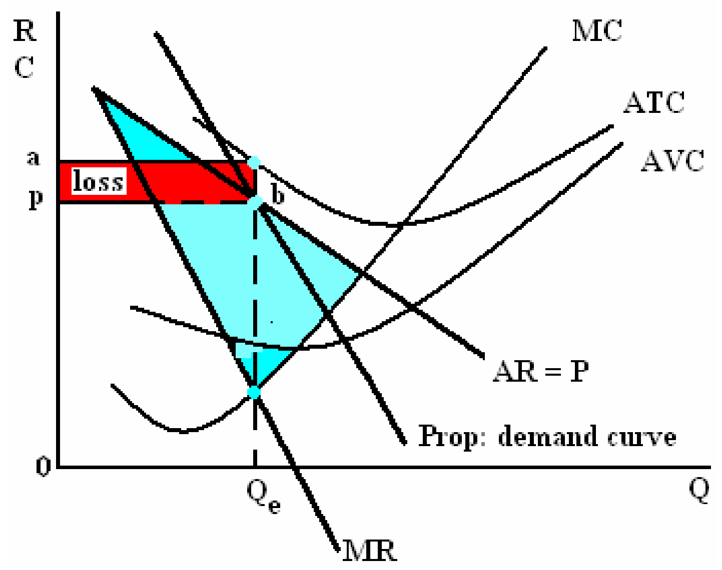
✓ **Normal profit:**

P = ATC



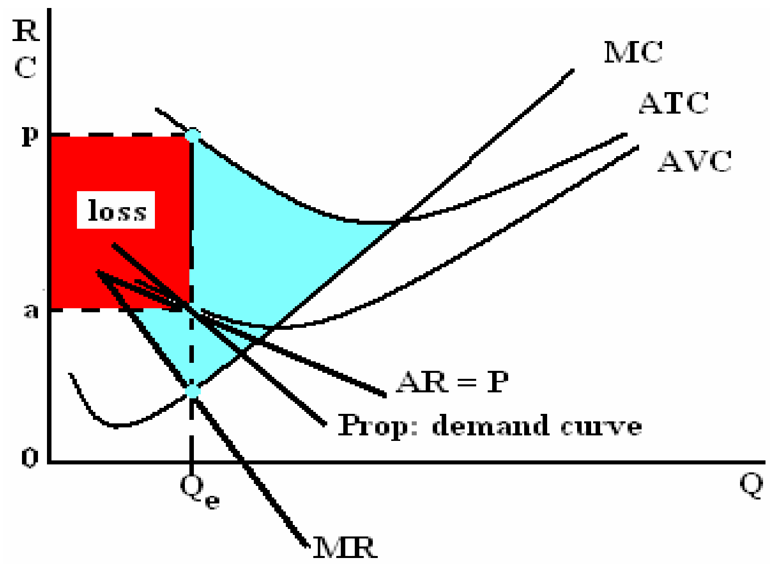
✓ **Normal loss of TFC:**

AVC < P < ATC



✓ **Loss of TFC:**

P = AVC

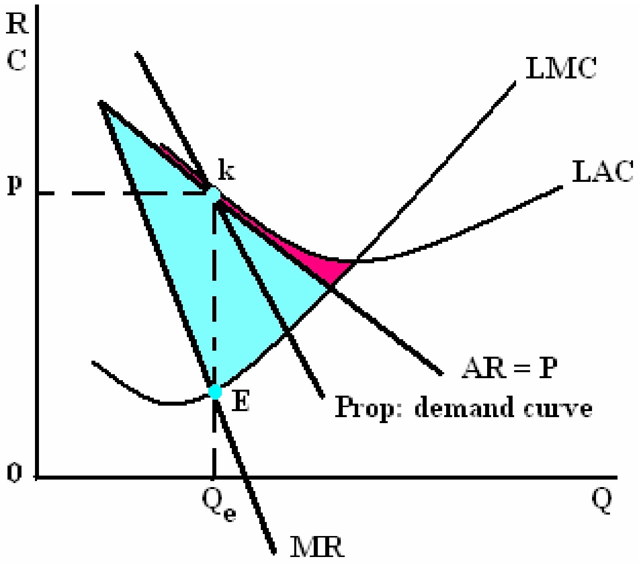


Monopolistic Competition

Reasons of normal profit in long-run:

- Price war among existing firms
- Entry of new firms

So, in monopolistic competition firm always earn the normal profit



Part-II

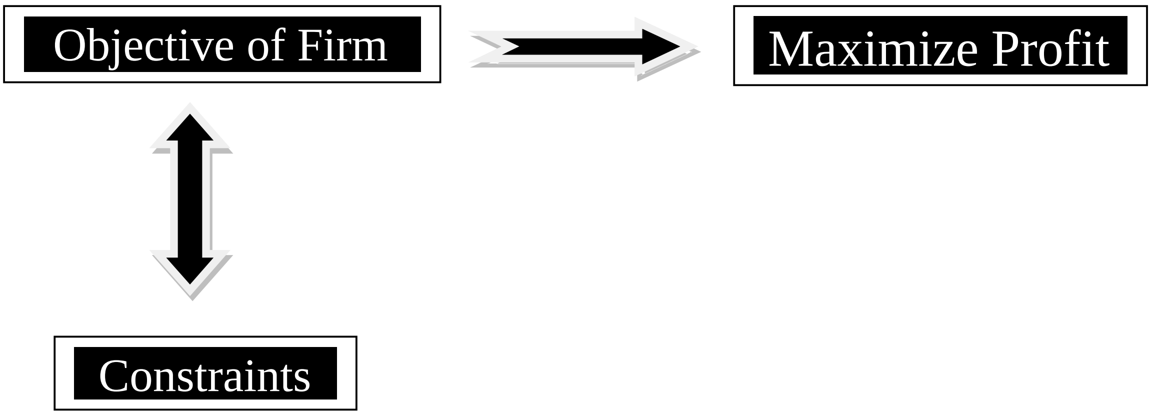
Microeconomics Analysis

Notes

Theory of production

Production analysis:

Ultimate objective of the firm is to maximization of profit, in order to maximize profit there are some constraints to follow



Constraints:

1- Technological constraints

2- Financial constraints

3- Market constraints

1- Technological constraints (T .C):

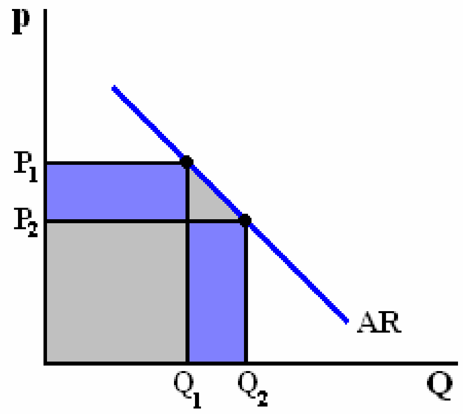
Those constraints which are related to the production possibility set or input requirement set i.e. fixed coefficient technology. Technological barriers can be seen through production function. “All those constraints which are liked with the production function are called T.C”.

2- Financial constraints (F .C):

"All those constraints which are linked with the budget of the firm as cost of production are called as F .C".

3- Market constraints (M .C):

"All those constraints which firm faced in getting/requiring input from the market and when firm selling their products in the market" i.e. perfect competition "firm can't change the price to increase the profit. All those constraints on price and output are market constraints. "Market constraints can be according to the market structures".



- High Prices
- Low Quality

- Low Prices
- High Quality

How we incorporate all three constraints in the profit equation

Profit equation:

$$\pi = P\,Q - C$$
$$= P.X_i - \sum_{i=1}^{n} X_i w_i$$

X_i = inputs (K, L)

W_i = input prices

Example: If there is perfect competition

$$P \quad [f(X_i)] \quad - \quad \sum_{i=1}^{n} X_i w_i$$

Reflecting financial constraints

Reflecting Technological

Reflecting marketing constraints

Example1: Output constraints
Cost minimizing firm

Example2: Budget constraints
Revenue maximizing firm

Behavior wise we have three Firms

1- Cost Minimizing firm

➔ When (Q) output is given:

Min: Cost
S.T: Output constraints
(ISO-Quant is given)

2- Revenue Maximizing firm

➔ When (C) cost is given:

Max: Revenue
S.T: Cost constraints
(Budget line is given)

3- Profit Maximizing firm

➔ When No restriction

There is no restriction on output and cost (unrestricted or free maximizing firm

Now we are going to discuss the constraints in detail

Technological constraints

Production: “Creation of Utility”
To change the shape of any thing and which thing has ability to satisfy human utility, “To transform an inferior goods to a superior one (its usefulness increases)”

Production process:

It is one through which inputs are transformed into output/final good

Production Process → Inputs → Final/Output

Classification of Inputs

Inputs are three types

i- Raw materials

These are inputs and which are transformed into output/final goods (combine with output and exhausted) i.e. Tree

ii- Energy resources

These were exhausted but do not combined in final goods/output i.e. electricity, gas

iii- Factors of Production

These inputs neither exhausted nor combined with final goods/outputs i.e. machinery, labor

Net output: Let there be any good "j" and apart of that used as input just like agriculture seeds

$Y_j^{(i)}$ "i" *is the amount of "j" which used as or served as input to produce* $Y_j^{(0)}$

Net out can be:

(1) *Positive* (+) $Y_j^{(i)} < Y_j^{(0)}$

(2) *Negative* (−) $Y_j^{(i)} > Y_j^{(0)}$

(3) *Zero* (0) $Y_j^{(i)} = Y_j^{(0)}$

➔ We would like to use such techniques through which we produce more output using lesser inputs

Production Plan:

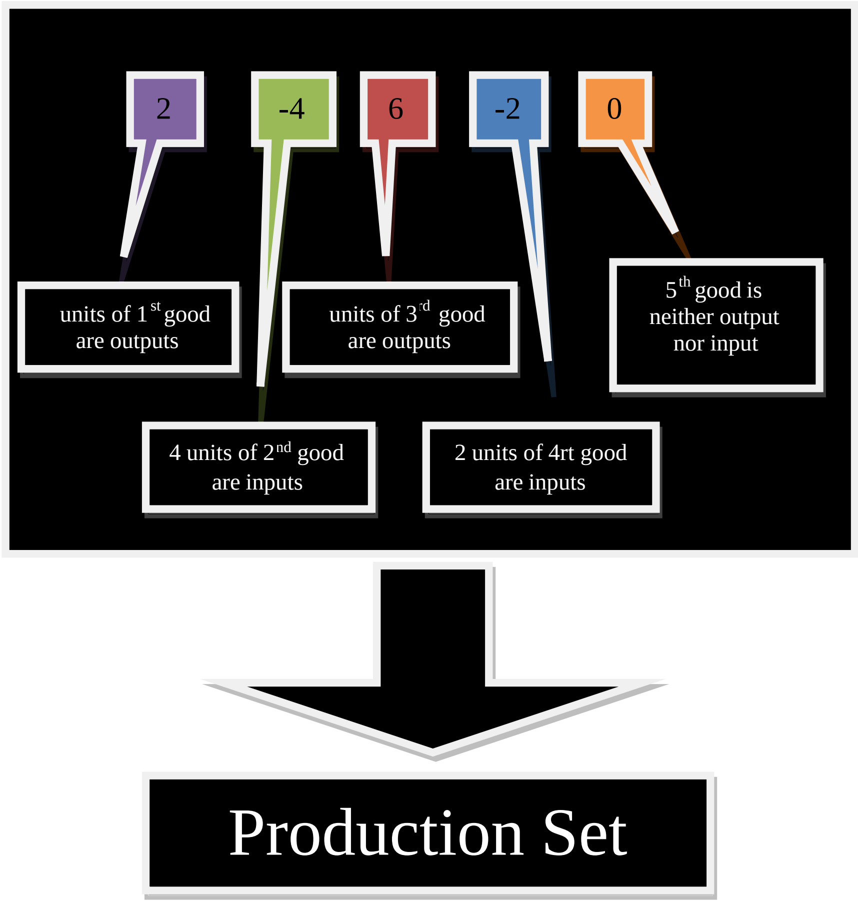
It is a set of different net outputs (only a set of feasible net outputs)

Notation:

$Y_j > 0$ *if* "j" *serves as output*

$Y_j < 0$ *if* "j" *serves as input*

$Y_j = 0$ *if* "j" *serves neither output nor input*



Input Requirement Set:
Combination of inputs which are required to get desired output

Example1: For output 1 we have two options

Option 1: 5 units of 1st input and 10 units of 2nd input

Option 2: 10 units of 1st input and 5 units of 2nd input (inputs are substitutes, this is valid)

$V\ (1)\ [(5,10)\quad (10,5)] \rightarrow \textit{Input requirement set}$

Example 2: To produce 2 units of output we have;

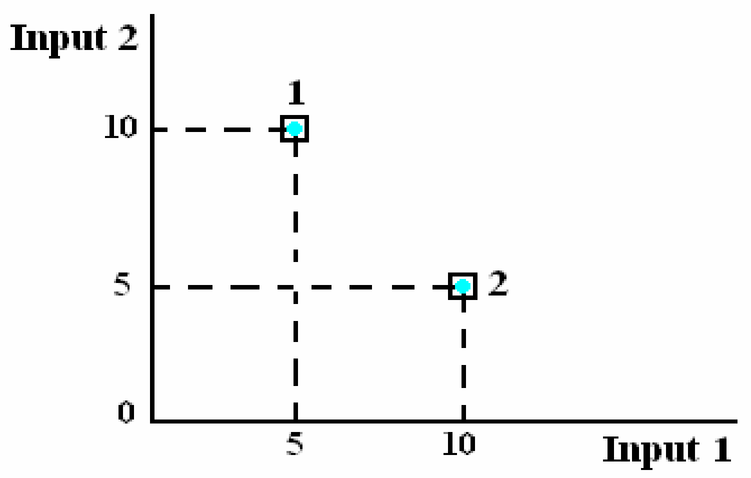
Option 1: 4 units of 1st input and 8 units of 2nd input

Option 2: 8 units of 1st input and 4 units of 2nd input

Option 3: 6 units of 1st input and 6 units of 2nd input

Mixed technology:
(8, 4) → 2 units of output
(4, 8) → 2 units of output
Or
(2, 4) → 1 unit of output

- Therefore we use mixed technology option to produce 2 units of output in option 3.



$$V(1) = [(5,10) \quad (10,5)]$$

. . .

. . .

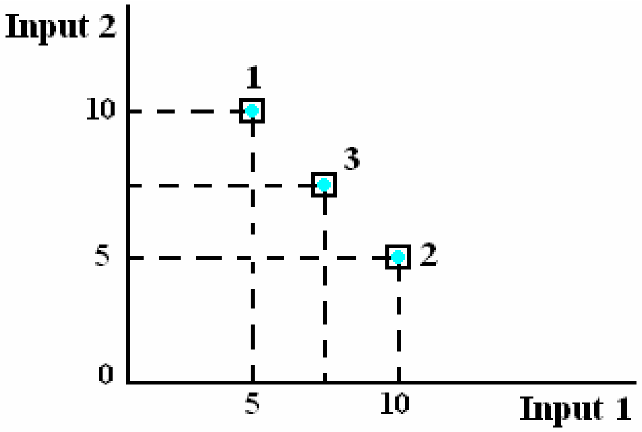
. . .

. . .

. . .

. . .

$$V(100) = [(500,1000) \quad (1000,500)]$$



$$V(2) = [(8,4) \quad (4,8) \quad (6,6)]$$

Properties of Production Set:

1- It is non-empty with reference to production

2- There is no free disposal (inputs are essential)

3- Define upper limit (given technology, inputs are not available)

4- If we have use of excessive inputs (it is possible definitely not feasible)

Le "X" be input requirement set to produce "Y" of output and X' > X then X' can also produce "Y" level of output

X = => V (Y)

X' = => V (Y) if X' ≥ X

It is monotonicity (it means more inputs can be exhausted to produce the given level of output

5- At given level of inputs less output can be produced

Similarly

If V (Y) = => X

Then V (Y') = => X such that Y' < Y

6- Production sets are convex

Convex set:

If A and B belongs to set K then K would be convex set if and only if linear combination of "A" and "B" also belongs to set "K"

⇨ If (A, B) Є to K

Then [λ (A) + (1 – λ) (B)] Є K

Example 1: A = {1, 2, 3………………… 1000}

Let take 10 and 100 and make linear combination of both

Then λ (10) + (1 – λ) (100)

Let λ = 0.50 then,

5 + 50 = Є A

If λ = 0.50 then 55 Є A

λ = 0.10 then 91 Є A

λ = 0.05 then 95.5 ∉ A

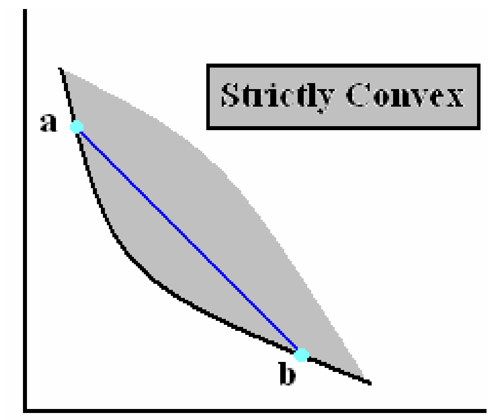
➔ Therefore "A" is not a convex set

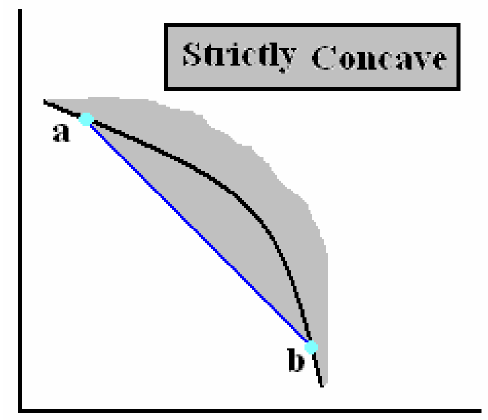
Example 2: B = {x/x Є Rational number V 0 ≤ x ≤ 100}

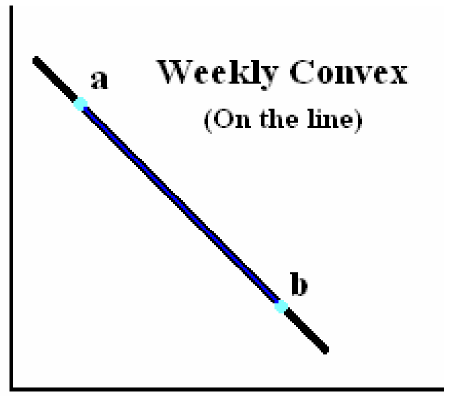
➔ It is convex set because rational set has fractional values

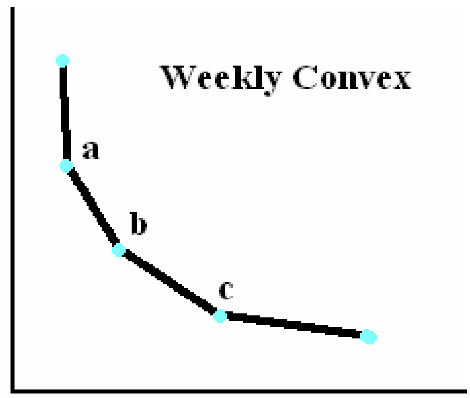
Example 3: C = {x/x 0 < x < 100 V 100 < x < 1000}

➔ If there is break in rational numbers then it is not convex set









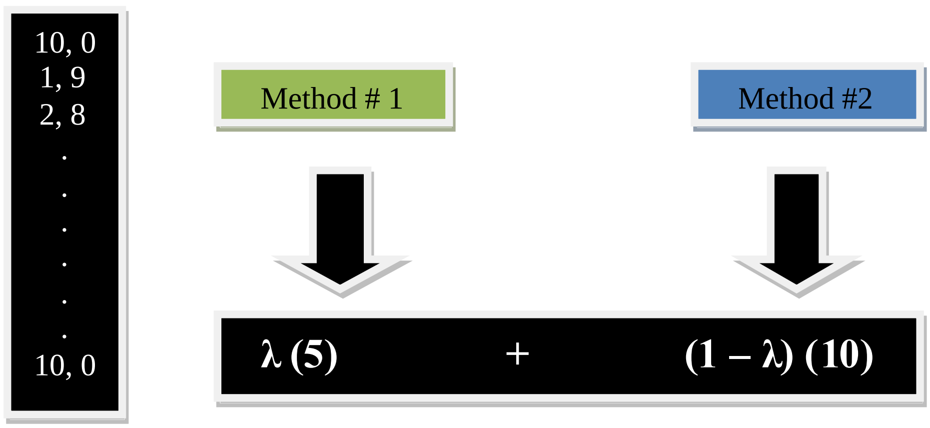
V (1) = [(5, 10)   (10, 5)]

.   .   .

.   .   .

V (10) = [(50, 100)   (100, 50)]

→ All linear combinations of inputs to produce a given level of outputs (even in fractions are possible)



Few concepts of describing technology:

- **ISO-Quant's:**

It is the different combinations of inputs that produce a given level of output; the ISO-quant gives all input bundles which produces given output level

- **Technical Rate of Substitution:**

- **Elasticity of Substitution:**

"The elasticity of substitution measures the relative responsiveness of the capital-labor ratio to given proportional change in the marginal rate of technical substitution of capital for labor".

The formula for elasticity of substitution (σ) is;

$$\sigma = \frac{\Delta(K/L)}{K/L} \div \frac{\Delta\, MRTS}{MRTS}$$

$$= \frac{\Delta(K/L)}{\Delta MRTS} \cdot \frac{MRTS}{K/L}$$

➔ Above the formula can be made more meaning full, recall that the marginal rate of technical substitution of capita for labor is the ratio of marginal product of labor to that capital

Let w, r and P denote the prices of labor, capital and output respectively, rearranging the VMP rule, we have;

$$MRTS = \frac{MP_L}{MP_K} = \frac{w/P}{r/P} = \frac{\omega}{r}$$

Hence in equilibrium, the elasticity of substitution may be written as;

$$\sigma = \frac{\Delta(K/L)}{\Delta(w/r)} \cdot \frac{w/r}{K/L}$$

$$\sigma = \frac{\Delta(K/L)}{K/L} \div \frac{\Delta(w/r)}{w/r}$$

➔ In this form, the elasticity of substitution shows the proportionate change in the capital-labor ratio induced by a given proportionate change in factor-price ratio

- **Returns to Scale:**

"If we are using some inputs (X) to produce some output (Y) and we are deciding to scale up/down all inputs by some amount ($t > 0$), what will happen to the level of output is as returns to scale".

1- Constant Returns to Scale:

"If we increase the inputs (X) by one unit and level of output will also be increased by same amount then it is called as *constant returns to scale*".

2- Increasing Returns to Scale:

"If we increase the inputs (X) by one unit and level of production/output will also increased by more than that amount, is called *increasing returns to scale*".

3- Decreasing returns to Scale:

"If we increase the inputs (X) by one unit and output/ level of production will increased by less than that amount, is called as *decreasing returns to scale*".

- **Elasticity of Scale:**

What happened to output if we increase all inputs (X) by some small amount "t", this is given by;

$$\frac{df(tx)}{dt}$$

➔ Now we convert this into elasticity measure to find:

$$\frac{df(tx)}{dt} \cdot \frac{t}{f(x)}$$

➔ We evaluate this measure at t = 1 to see what the elasticity of scale is at X;

$$e(x) = \frac{df(tx)}{dt} \cdot \frac{1}{f(x)} \Big|_{t=1}$$

The elasticity of scale [e(x)] measures *"the percentage increase in output due to a percentage increase in scale (inputs)"*.

$\rightarrow$ $e(x)$ is greate, equal or less than 1

- **Homogeneous and homothetic technologies:**

Production function

There are two broader groups of production functions

A- With respect to Inputs

- **i.** Short-period Production functions[1]
- **ii.** Long-period Production functions[2]

B- With respect to Technology

- **i-** Flexible Production functions[3]
- **ii-** Rigid Production functions[4]

There are some Production Functions:

1- Leontief Production Function/fixed coefficient production function (rigid)

2- Linear Production Function(rigid)

3- Cob-Douglas Production Function (rigid)

4- CES (**C**onstant Elasticity of Substitution) Production Function

5- Translog Production Function (flexible)

1- If there is no limitation or availability restriction on inputs [Y = f(L, K)]

2- If there is restriction over any input [$Y = f(L, \bar{K})$]

3 - If we have flexibility to use whatever the technology mix or formula for inputs

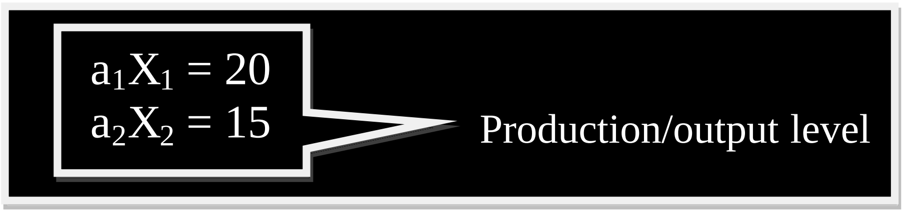
4- If there is no flexibility to use whatever technology mix means there is rigidity in input usage

✓ ***Leontief Production Function/fixed coefficient production function (rigid):***

In this production function inputs are used in fixed proportion, we can not change the proportion.

Mathematically:

$$Y = \min[a_1X_1, \quad a_2X_2]$$



Example 1: Let

$a_1 = 10$ $\quad a_2 = 12.5$

$X_1 = 5$ $\quad X_2 = 6$

There is input constraint on X_1 input's availability

Therefore $\quad a_1X_1 = 50, a_2X_2 = 75$

$a_1X_1 < a_2X_2$

:. $Y = a_1X_1$ $\quad$ [2 units of X_2 are extra]

$Y = 50$ (feasible)

Example 2: Let

$a_1 = 10$ $\quad a_2 = 12.5$

$X_1 = 5$ $\quad X_2 = 2$

There is input constraint on X_2 input's availability

Therefore $\quad a_1X_1 = 50, a_2X_2 = 25$

$a_1X_1 > a_2X_2$

:. $Y = a_2X_2$ $\quad$ [2.5 units of X_1 are extra]

$Y = 25$ (feasible)

Example 3: Let

$a_1 = 10$ $\quad a_2 = 12.5$

$X_1 = 5$ $\quad X_2 = 4$

There is no input constraint on input's availability, means effective use of both inputs

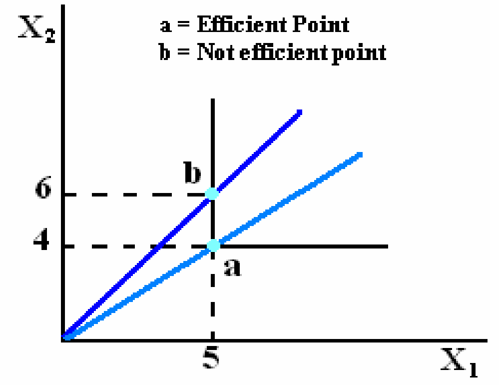
Therefore $\quad a_1X_1 = 50, a_2X_2 = 50$

$a_1X_1 = a_2X_2$

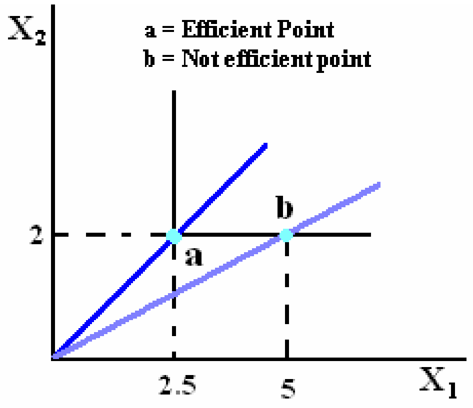
:. $Y = 50$ (feasible)

Graphically:

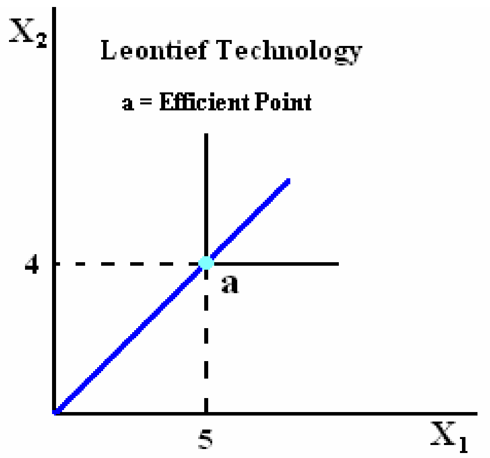
Example 1:



Example 2:



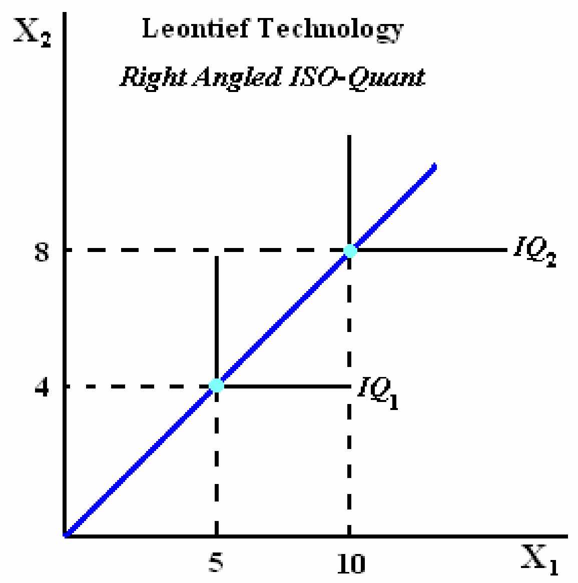
Example 3:



- Leontief feasible point is *right angled ISO-Quant (kinked ISO-Quant)*, in 3rd example;

$$a_1X_1 = a_2X_2$$

$$\frac{a_1}{a_2} = \frac{X_2}{X_1}$$



- Leontief technology is one in which inputs are used in fixed proportion in Leontief function

Properties of Leontief Production Function:

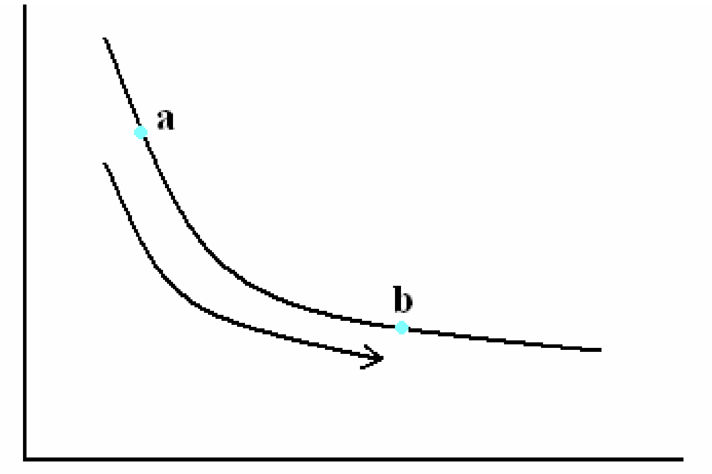
1- It is constant returns to scale

2- It violates monotonicity in inputs (input increases then output increases), it conditionally satisfy monotonicity in inputs(if one input increases with following proportion and production will never increases)

3- Elasticity of Substitution is zero

$$\sigma = \frac{Proportionate\ change\ in\ input\ ratio}{proportionate\ change\ in\ slope\ of\ ISO - Quant}$$

$$\sigma = \frac{Proportionate\ change\ in\ input\ ratio}{proportionate\ change\ in\ MRTS}$$



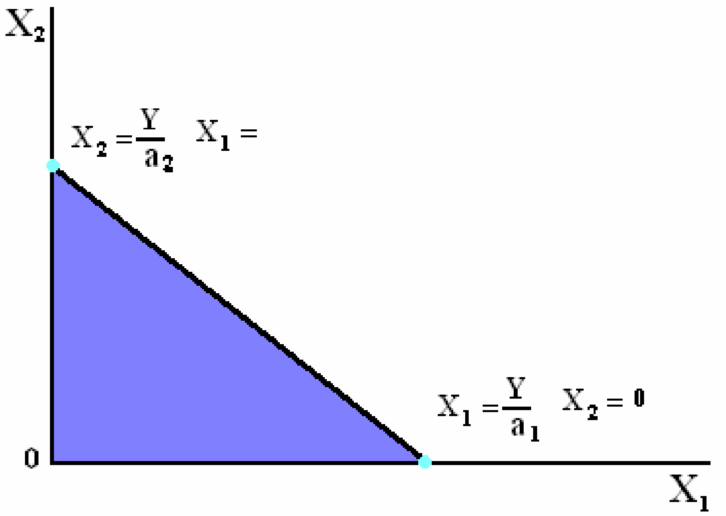
- It is measure of degree of substitution between two inputs due to this a result of change in price ratio;

$$\sigma = \frac{Proportionate\ change\ in\ Price\ ratio(=0)}{proportionate\ change\ in\ MRTS}$$

- Perfectly complimentary inputs

✓ *Linear Production Function(rigid):*

$$\bar{Y} = a_1X_1 + a_2X_2$$



$$slope = \frac{-a_1}{a_2} \quad as\ like\ \frac{-P_1}{P_2}$$

- Perfect substitutes
- Elasticity of substitution is infinity

Cob-Douglas Production Function:

Basic general form of cob-Douglas function is;

$$Q = A(L)^{\alpha}(K)^{\beta} \qquad \alpha > 0, \beta > 0$$

General form of n-term;

$$Q = A(X_1)^{\alpha_1}(X_2)^{\alpha_2}(X_3)^{\alpha_3} \dots \dots \dots \dots \dots \dots . . (X_n)^{\alpha_n}$$

Original form of Cob0Douglas Function

$$Q = A(L)^{\alpha}(K)^{1-\alpha}$$

Properties of Cob-Douglas Function:

1- It is essential to use both inputs (no corner solution)

- partial elasticity of output with respect to labor:

$$\varepsilon_{Q_L} = \frac{\partial Q}{\partial L} \cdot \frac{L}{Q}$$

As we know typical form of Cob-Douglas function;

$$Q = A(L)^{\alpha}(K)^{\beta}$$

$$\therefore \frac{\partial Q}{\partial L} = A\alpha L^{\alpha-1}K^{\beta}$$

$$= \alpha L^{-1}(AL^{\alpha}K^{\beta})$$

$$\therefore \frac{\partial Q}{\partial L} = \frac{\alpha Q}{L}$$

As we know that

$$\varepsilon_{Q_L} = \frac{\partial Q}{\partial L} \cdot \frac{L}{Q}$$

$$= \frac{\alpha Q}{L} \cdot \frac{L}{Q}$$

$$\therefore \varepsilon_{Q_L} = \alpha$$

- Similarly, partial elasticity of output with respect to capital:

$$\varepsilon_{Q_K} = \frac{\partial Q}{\partial K} \cdot \frac{K}{Q}$$

As we have typical form of Cob-Douglas function;

$$Q = A(L)^{\alpha}(K)^{\beta}$$

$$\therefore \frac{\partial Q}{\partial K} = A\beta L^{\alpha}K^{\beta-1}$$

$$= \beta(AL^{\alpha}K^{\beta})K^{-1}$$

$$\therefore \frac{\partial Q}{\partial K} = \frac{\beta Q}{K}$$

As we know that

$$\varepsilon_{Q_K} = \frac{\partial Q}{\partial K} \cdot \frac{K}{Q}$$

$$= \frac{\beta Q}{K} \cdot \frac{K}{Q}$$

$$\therefore \varepsilon_{Q_K} = \beta$$

$$\alpha = \frac{Percentage\ change\ in\ Output}{1\%\ chnage\ in\ labor}$$

$$\beta = \frac{Percentage\ change\ in\ Output}{1\%\ chnage\ in\ capital}$$

- Where (α + β) measures the returns to scale

Let α = 0.5, β = 0.5 } α + β = 1

- if α + β = 1 that indicates constant returns to scale
- if α + β > 1 that indicates increasing returns to scale
- if α + β < 1 that indicates decreasing returns to scale

As we know that

$$\frac{\partial Q}{\partial L} = \frac{\alpha Q}{L} => MP_L$$

$$\therefore MP_L = \frac{\alpha Q}{L}$$

$$=> AP_L = \frac{Q}{L}$$

where

$$MP_L = \alpha(\frac{Q}{L})$$

$$MP_L = \alpha \cdot AP_L$$

$$\therefore \alpha = \frac{MP_L}{AP_L}$$

Similarly for capital;

$$\frac{\partial Q}{\partial K} = \frac{\beta Q}{K} => MP_K$$

$$\therefore \quad MP_K = \frac{\beta Q}{K}$$

$$=> AP_K = \frac{Q}{K}$$

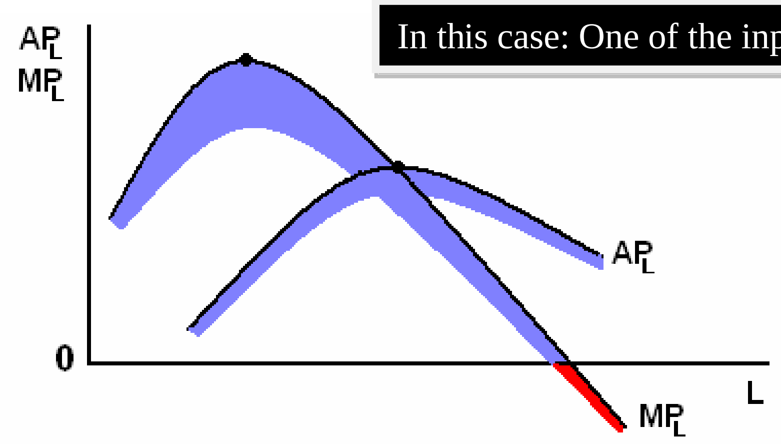
$$where \quad MP_K = \beta(\frac{Q}{K})$$

$$MP_K = \beta \cdot AP_K$$

$$\therefore \quad \beta = \frac{MP_K}{AP_K}$$

As we know that

In this case: One of the inputs is fixed



$$if \quad MP_L = AP_L$$

$$then \; \alpha = 1$$

$$let \;\; \alpha + \beta = 1$$

$$\frac{MP_L}{AP_L} + \frac{MP_K}{AP_K} = 1$$

2- The coefficient of inputs (α, β) are partial elasticities of output with respect to inputs

3- The sum of coefficients (α + β) is the measure of returns to scale

4- Any coefficient is the ratio of its marginal and average products

- If it is case of perfect competition, how wage rate is determined?

$$w = VMP_L$$

$$w = (MP_L)(P_Q)$$

As we know that

$$MP_L = \frac{\alpha Q}{L}$$

$$\therefore \quad w = \left(\frac{\alpha Q}{L}\right)(P_Q)$$

$$w \cdot L = (\alpha Q)(P_Q)$$

$$\therefore \quad w \cdot L = Total\ Labor\ Cost$$
$$\therefore (P_Q)Q = Total\ revenue$$

$$\alpha = \frac{w \cdot L}{(P_Q)Q}$$

$$\therefore \quad \alpha = \frac{Total\ Labor\ Cost}{Total\ revenue}$$

If there are two inputs then;

Total Cost = Total Labor Cost + Total Capital Cost

$$TC = w.L + r.K$$

- Limitation of Cob-Douglas function:

$$if\ \alpha + \beta = \frac{w.L}{PQ} + \frac{r.K}{PQ}$$

$$= \frac{w.L + r.K}{PQ}$$

$$\therefore \quad \alpha + \beta = \frac{Total\ Cost}{Total\ Revenue}$$

- It is rigidity of Cob-Douglas production function that a firm always runs normal profit

5- Elasticity of Substitution is equal to unity (one).

Proof: